Australian Rainfall & Runoff

A GUIDE TO FLOOD ESTIMATION

BOOK 4 - CATCHMENT SIMULATION FOR DESIGN FLOOD ESTIMATION

Version 4.2

Australian Government
ENGINEERS AUSTRALIA

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PREFACE

Since its first publication in 1958, Australian Rainfall and Runoff (ARR) has remained one of the most influential and widely used guidelines published by Engineers Australia (EA). The 3rd edition, published in 1987, retained the same level of national and international acclaim as its predecessors.

With nationwide applicability, balancing the varied climates of Australia, the information and the approaches presented in Australian Rainfall and Runoff are essential for policy decisions and projects involving:

- infrastructure such as roads, rail, airports, bridges, dams, stormwater and sewer systems;
- town planning;
- mining;
- developing flood management plans for urban and rural communities;
- flood warnings and flood emergency management;
- operation of regulated river systems; and
- prediction of extreme flood levels.

However, many of the practices recommended in the 1987 edition of ARR have become outdated, and no longer represent industry best practice. This fact, coupled with the greater understanding of climate and flood hydrology derived from the larger data sets now available to us, has provided the primary impetus for revising these guidelines. It is hoped that this revision will lead to improved design practice, which will allow better management, policy and planning decisions to be made.

One of the major responsibilities of the National Committee on Water Engineering of Engineers Australia is the periodic revision of ARR. While the NCWE had long identified the need to update ARR it had become apparent by 2002 that even with a piecemeal approach the task could not be carried out without significant financial support. In 2008 the revision of ARR was identified as a priority in the National Adaptation Framework for Climate Change which was endorsed by the Council of Australian Governments.

In addition to the update, 21 projects were identified with the aim of filling knowledge gaps. Funding for Stages 1 and 2 of the ARR revision projects were provided by the now Department of the Environment. Stage 3 was funded by Geoscience Australia. Funding for Stages 2 and 3 of Project 1 (Development of Intensity-Frequency-Duration information across Australia) has been provided by the Bureau of Meteorology. The outcomes of the projects assisted the ARR Editorial Team with the compiling and writing of chapters in the revised ARR. Steering and Technical Committees were established to assist the ARR Editorial Team in guiding the projects to achieve desired outcomes.

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Status of this document

This document is a living document and will be regularly updated in the future.

In development of this guidance, and discussed in Book 1 of ARR 1987, it was recognised that knowledge and information availability is not fixed and that future research and applications will develop new techniques and information. This is particularly relevant in applications where techniques have been extrapolated from the region of their development to other regions and where efforts should be made to reduce large uncertainties in current estimates of design flood characteristics.

Therefore, where circumstances warrant, designers have a duty to use other procedures and design information more appropriate for their design flood problem. The Editorial team of this edition of Australian Rainfall and Runoff believe that the use of new or improved procedures should be encouraged, especially where these are more appropriate than the methods described in this publication.

Care should be taken when combining inputs derived using ARR 1987 and methods described in this document.

Change Log

Version 4.2 - Climate Change Chapter Update

In late 2022 the Australian Government Department of Climate Change, Energy, the Environment and Water in partnership with Engineers Australia commenced an 18 month project to update the climate change considerations chapter of the Australian Rainfall and Runoff guidelines (Chapter 6, Book 1) to incorporate the most recent and relevant climate science and projections. The project involved the undertaking of a rigorous literature review of hydroclimatology under climate change relevant to design flood estimation, which was peer reviewed and published in a leading international journal. The findings were used to draft practical flood guidance which was finalised after an extensive process of review and feedback by industry. Funding for this project was received from National Emergency Management Agency under the Disaster Risk Reduction Package. The project report was adapted to replace Book 1 chapter 6.

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This version updates Book 1 Chapter 6 to reflect updates in climate science as discussed above. While no other chapters have been updated some minor amendments were made to remove inconsistencies with the new chapter. FAQs relating to the update are available https://arr.ga.gov.au/contact-us.

Key updates in Version 4.2

Update	Version 4.2
Book 1	Book 1 Chapter 6 Climate change updated
Guideline formats	PDF Web-based version Epub version
User experience	FAQs added to Geoscience Australia Website
Climate change	Reflected best practice as of 2024 and IPCC 6
Other Minor Changes	List the minor changes to the following chapters for consistency Book 1 Chapter 4 Section 15.1 Book 1 Chapter 4 Section 16.1 Book 1 Chapter 5 Section 10.4 Book 2 Chapter 1 Section 3 Book 2 Chapter 3 Section 3 Book 6 Chapter 5 Section 5 Book 8 Chapter 7 Section 7 Book 9 Chapter 6 Section 4.2 Book 9 Chapter 6 Section 4.6

ARR 2019 (now Version 4.1)

Geoscience Australia, on behalf of the Australian Government, asked the National Committee on Water Engineers (NCWE) - a specialist committee of Engineers Australia - to continue overseeing the technical direction of ARR. ARR's success comes from practitioners and researchers driving its development; and the NCWE is the appropriate organisation to oversee this work. The NCWE has formed a sub-committee to lead the ongoing management and development of ARR for the benefit of the Australian community and the profession. The current membership of the ARR management subcommittee includes Mark Babister, Robin Connolly, Rory Nathan and Bill Weeks.

The ARR team have been working hard on finalising ARR since it was released in 2016. The team has received a lot of feedback from industry and practitioners, ranging from substantial feedback to minor typographical errors. Much of this feedback has now been addressed. Where a decision has been made not to address the feedback, advice has been provided as to why this was the case.

A new version of ARR is now available. ARR 2019 is a result of extensive consultation and feedback from practitioners. Noteworthy updates include the completion of Book 9, reflection of current climate change practice and improvements to user experience, including the availability of the document as a PDF.

Key updates in ARR 2019

Update	ARR 2016	ARR 2019
Book 9	Available as “rough” draft	Peer reviewed and completed
Guideline formats	Epub version Web-based version	Following practitioner feedback, a pdf version of ARR 2019 is now available
User experience	Limited functionality in web-based version	Additional pdf format available
Climate change	Reflected best practice as of 2016 Climate Change policies	Updated to reflect current practice
PMF chapter	Updated from the guidance provided in 1998 to include current best practice	Minor edits and reflects differences required for use in dam studies and floodplain management
Examples		Examples included for Book 9
Figures		Updated reflecting practitioner feedback

As of May 2019, this version was considered to be final.

ARR 2016 (now Version 4.0)
Released July 2016

BOOK 4

Catchment Simulation for Design Flood Estimation

Catchment Simulation for Design Flood Estimation

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Chapter 1. Introduction

James Ball, Rory Nathan

Chapter Status	Final
Date last updated	14/5/2019

1.1. Simulation of Design Flood Hydrographs

There are many problems where design flood characteristics other than the peak flows are required. Most commonly this involves problems where the volume of the flood hydrograph has a dominant influence on the design objective of interest. Typical problems include the design of urban stormwater drainage systems where it is necessary to size on-site detention storages, the estimation of maximum surcharge levels in detention basins and dams, and the derivation of design flood levels in situations where the floodplain can store an appreciable proportion of the event. It may also be necessary to characterise the rate of rise of selected events for flood warning purposes. The assessment of such problems requires the simulation of complete design flood hydrographs, where it may be necessary to give particular attention to the rate of rise and the total volume of the hydrograph. If design interest is focused on a location in the catchment that is materially influenced by hydraulic controls (as in many urban catchments), then it is likely that it will be necessary to model the catchment as a “system” using an integrated combination of hydrologic and hydraulic modelling (therefore use of a catchment modelling system, Book 7). Conversely, if the point of interest is largely uninfluenced by hydraulic controls (as in many rural catchments or trunk drainage networks in urban areas) then it should be sufficient to model the catchment with a hydrologic model.

The simulation of design flood hydrographs is most easily undertaken using rainfall data. While the methods presented in Book 3 provide useful independent estimates on the peak of simulated hydrographs, additional information is required to check the shape and volume of design flood hydrographs. Rainfall-based methods involve the transformation of rainfalls into a selected flood characteristic:

- event-based models transform probabilistic bursts of rainfall to corresponding estimates of floods; and
- continuous simulation models transform a time series of rainfall into probabilistic flood estimates

Hydraulic models are then used to simulate flood levels from hydrologic (and/or rainfall) inputs. The challenge with these methods is how to achieve probability neutrality, that is how to ensure that the method used to transform rainfalls into design floods is undertaken in a fashion that minimises bias in the resulting exceedance probabilities.

The guidance in this Book is focused on the conceptual frameworks used to derive design flood hydrographs, rather than on the models used to transform rainfall into runoff. Detailed guidance on the different types of hydrologic models is provided in Book 5, and guidance on the hydraulic modelling of runoff through the catchment is provided in Book 6. Guidance on the application of catchment modelling systems and the interpretation of the results obtained is provided in Book 7.

1.2. Difference Between Historic and Design Flood Simulations

It is worth noting that there is a considerable difference in the modelling approaches required to simulate historic (or observed) and design floods. Although the same mathematical procedures (and software) may be involved in both, the simulation objectives and modelling considerations are markedly different.

Estimation of a design flood involves the derivation of the relationship between the magnitude and probability of a given flood characteristic. The objective of the analysis is to provide information for risk-based planning or design purposes. Such information can be extended to provide standards-based estimates such as the Probable Maximum Flood, but the simulation objective is still to assess the performance of a system under particular set of loading conditions.

In contrast, the modelling required to simulate floods using historic or forecast rainfalls involves quite different considerations. The challenge of selecting data inputs and analysis frameworks to estimate the exceedance probability of a particular outcome is replaced by the difficulty of preparing data to reflect conditions specific to a particular event in time. The focus of the analysis might be on simulating a particular historic flood, or it may involve assessment of the antecedent conditions and forecast rainfalls associated with a flood forecast required at a particular point in time.

Identical models might be used to simulate the response of the catchment under historic or design conditions. The essential difference is that design flood simulations are undertaken to derive the best estimate of the relationship between flood magnitude and exceedance probability, whereas simulation of actual floods represent the best estimate of flood characteristics for a particular point in time. The Guidance provided in Book 6 and Book 7 is equally applicable to models used for the simulation of design or actual floods. The guidance provided in this Book is specific to the issues involved in assigning an exceedance probability to flood characteristics.

1.3. Scope

The scope of this Book is largely on the simulation frameworks used to derive design floods. Particular attention is paid to those hydrologic processes that most influence flood magnitude, and on the approaches required to estimate the exceedance probability of a flood characteristic of interest. Guidance is included on the treatment of joint probability, as the explicit analysis of the way in which factors combine to influence the frequency of floods is an essential consideration in the estimation of design floods. Worked examples are provided to assist practitioners apply the guidance to typical real-world problems.

1.4. Outline

This book is structured as follows:

- Book 4, Chapter 2 provide a broad description of the runoff processes that contribute to streamflows (interception, depression storage, infiltration, interflow, and groundwater contributions), and identifies those components of most relevance to flood generation;

- Book 4, Chapter 3 describes three different approaches to event-based modelling (simple, ensemble, and Monte Carlo methods) and describes how increasing levels of sophistication can be used to minimise bias in the transformation of design rainfalls into design floods; Book 4, Chapter 3 also describes the use of continuous (and hybrid) simulation approaches to flood estimation, and summarises the strengths and weaknesses of the various approaches;

- Book 4, Chapter 4 introduces the generic nature of joint probability requirements; it covers the factors involved in the transformation of rainfall to runoff, and other factors (e.g. initial reservoir level or tide levels) that may influence the design performance of interest; and

- Book 4, Chapter 3 and Book 4, Chapter 4 include worked examples that illustrate application of the techniques to practical problems.

Chapter 2. Hydrologic Processes Contributing to Floods

Anthony Ladson, Rory Nathan

Chapter Status	Final
Date last updated	14/5/2019

2.1. Introduction

This chapter outlines the hydrologic processes that contribute to floods including a review of runoff generation, baseflow contributions to flood flow, flow routing and losses. The chapter concludes with a discussion of the conceptualisation of these processes in models and case studies of floods in tropical and temperate rural catchments, and in urban areas.

Under Australian conditions, the ultimate cause of the large streamflows that result in floods is usually rainfall. Other causes, such as melting of snow and ice, are less important in our temperate climate. In places storm surge may combine with stream flows to cause flooding as discussed in Book 6, Chapter 5.

The link between rainfall and streamflow is mediated by a number of processes (Figure 4.2.1). Rainfall landing on the catchment surface can be converted to runoff in different ways that depend on infiltration capacity and whether soils are saturated. Four runoff processes are discussed in Book 4, Chapter 2, Section 2: those relating to infiltration excess, saturation excess, sub-surface stormflow and impervious area runoff. Typically, only a small proportion of rainfall will become streamflow with the rest being evaporated perhaps after being intercepted by vegetation, stored in surface depressions or infiltrated to become soil moisture or groundwater. Some groundwater may contribute to floods via baseflow (refer to Book 5, Chapter 4).

There are particular conditions that can lead to high streamflow, and flooding. A 'wet' catchment means reduced losses so that a greater proportion of rainfall will be converted to runoff. A catchment could be wet up by a long period of low intensity rainfall, particularly when evaportranspiration is low, such as in winter. A short burst of high intensity rainfall can lead to flooding if there are limited opportunities for rain to be lost. This is particularly the case in catchments where impervious surfaces and piped drainage systems link runoff to streams.

Figure 4.2.1 summarises the physical processes that can lead to floods, but floods can also be considered stochastic events caused by the random simultaneous occurrence of unusual conditions. The stochastic nature of flooding was illustrated in Book 1, Chapter 3 where it was shown that flood peaks resulting from 1% AEP rainfalls ranged in magnitude from 500 m^3/s to 2000 m^3/s. The cause of this disparity in response is random variation in catchment processes, such as interception and storage, and other factors such as the spatial and temporal patterns of rainfall. The series of peak flows at a gauge are manifestations of the joint probability of these random processes.

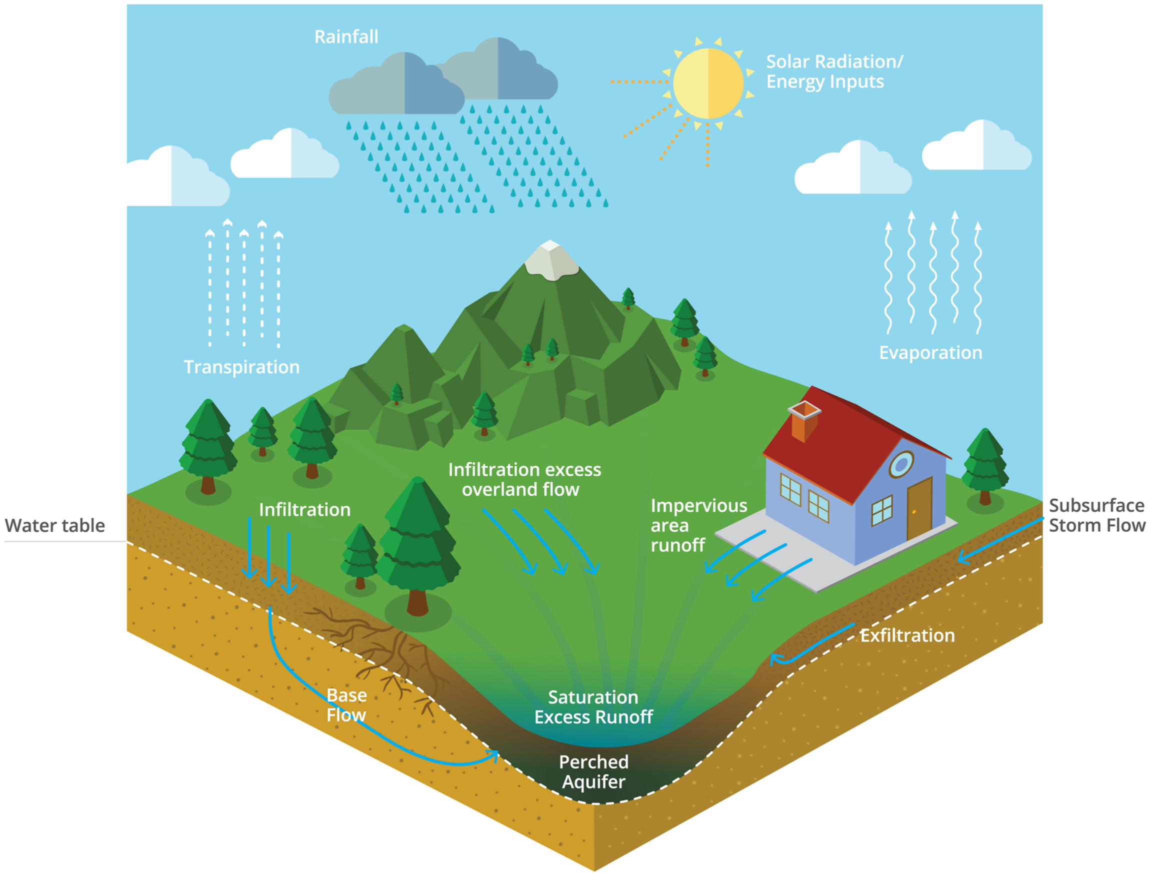

Figure 4.2.1. Catchment and Runoff Generation Processes

2.2. Runoff Generation

This section outlines some of the key runoff generation processes that can lead to floods. In particular, the following topics are addressed:

- Infiltration excess runoff;
- Saturation excess runoff;
- Variable source areas;
- Partial area runoff;
- Subsurface storm flow; and
- Impervious area runoff.

Here we are focussing on quickflow and the mechanisms that rapidly convert rainfall to streamflow and so cause a flood hydrograph. Book 4, Chapter 2, Section 3 briefly discusses the slower process of baseflow along with losses and flow routing (Figure 4.2.2).

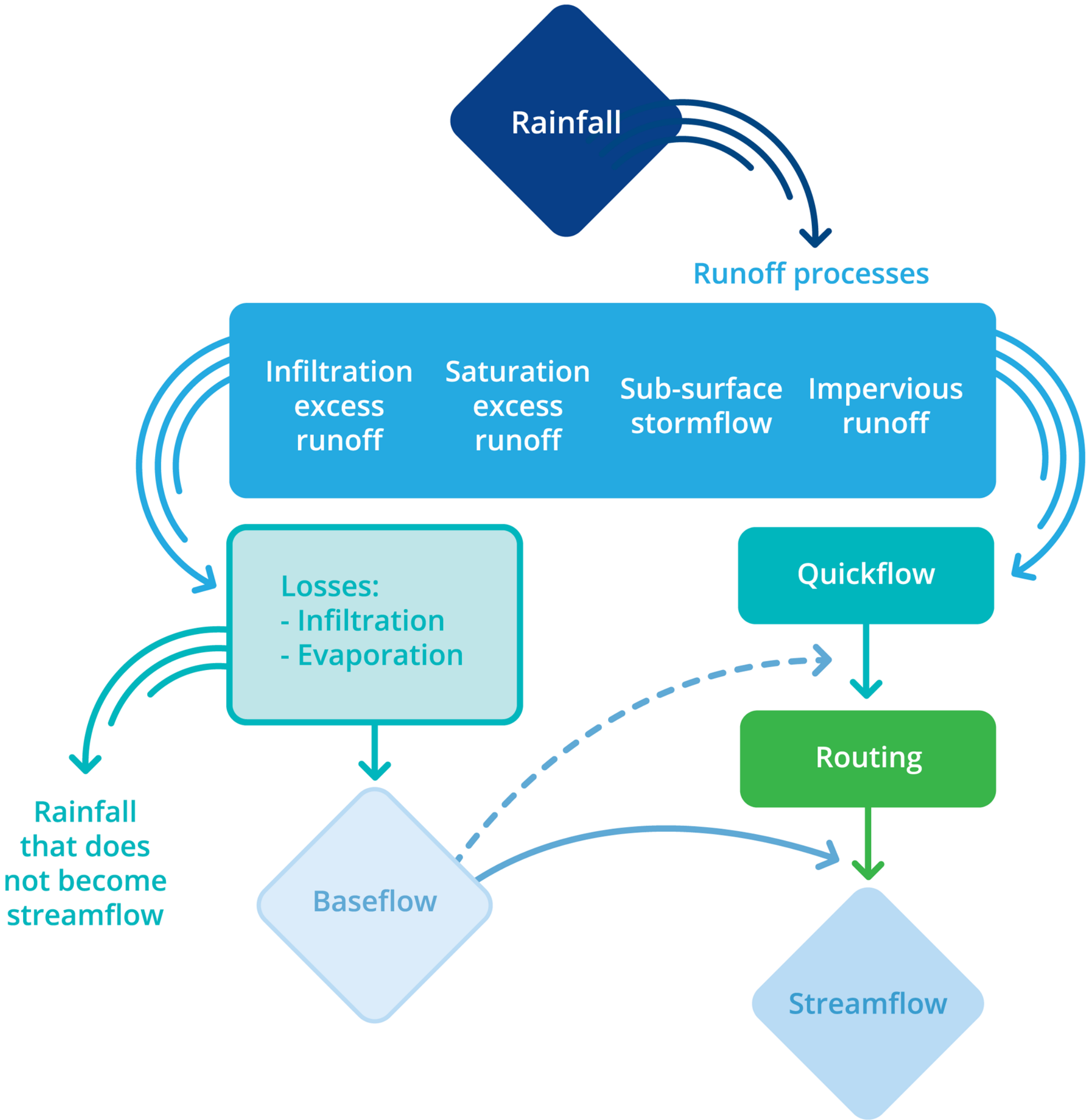

Figure 4.2.2. Simplified Description of the Process of Converting Rainfall to Runoff and Streamflow

2.2.1. Infiltration Excess Runoff

Once rainfall on a catchment reaches the soil surface, some will infiltrate into the soil. The infiltration rate, the rate at which water enters the soil, depends on:

- the rate at which water is supplied to the soil surface; and
- the infiltration capacity which is the maximum rate at which water can enter the soil.

If the rainfall rate (mm/hr) is greater than the infiltration capacity, water will pond at the soil surface; if the ground is sloping, then water will runoff. Runoff produced in this way is called infiltration excess overland flow, or Hortonian[1] overland flow. Hortonian overland flow can provide a rapid pathway for water to be converted from rainfall to runoff. Hortonian flow is likely to contribute to floods when catchment surfaces have low infiltration capacity, when there is intense rainfall and where there is a rapid mechanism for runoff to reach a stream.

2.2.2. Saturation Excess Runoff, Variable Source Areas and Partial Area Runoff

If soil becomes saturated, from rising soil moisture or because of flow from up-slope, then no additional rainfall can infiltrate. Any rainfall striking the saturated soil surface will be converted to saturation excess runoff. These saturated regions of a catchment are referred to as source areas.

Usually there are some areas within a catchment that are wetter than others. Areas along valleys and adjacent to streams may remain saturated for long periods with up-slope areas being dryer. Saturated areas enlarge and contract with the seasonal wetting and drying of a catchment. Saturated areas may expand during a storm and then shrink once rainfall ceases. As the amount of saturated area changes so does the source area contributing to runoff.

The concept of partial area runoff arises because only part of a catchment may be saturated and this area may be the only contributor to streamflow (Dunne and Black, 1970). Saturation excess runoff can contribute to floods when source areas are large and convert intense rainfall to runoff that flows directly to streams.

2.2.3. Impervious Area Runoff

Some natural catchments may contain impervious areas, such as rocky outcrops. Urbanisation leads to catchments being covered with roofs, roads, car parks and other impervious surfaces. A large proportion of rainfall landing on these surfaces is converted to runoff as there are few opportunities for rainfall to be intercepted and lost. Consequently, urbanisation leads to a large increase in runoff volume, flood frequency and magnitude. The hydrologic impacts of urbanisation have been quantified in a wide range of studies. Urbanisation causes up to a 10-fold increase in peak flows of floods in the range of 1 to 4 Exceedances per Year (EY), with diminishing impacts on larger floods (Tholin and Keifer, 1959; ASCE, 1975; Espey and Winslow, 1974; Hollis, 1975; Cordery, 1976; Ferguson and Suckling, 1990). Runoff in urban streams responds more rapidly compared to rural catchments (Mein and Goyen, 1988) and flow volumes increase (Harris and Rantz, 1964; Cordery, 1976; Ferguson and Suckling, 1990). Hydrologic impacts of urbanisation are discussed in Book 4, Chapter 2, Section 7 and in Book 9.

2.2.4. Subsurface Storm Flow

Subsurface flows can be an important source of flood runoff in areas with steep slopes, conductive soils and where the soil profile becomes saturated so that water can move through large pores. In many forested catchments surface runoff is rare. Soil infiltration rates are never exceeded by rainfall and confined streams limit opportunities for formation of saturated source areas. Instead, given appropriate soil conditions, water may be rapidly transferred down-slope as subsurface flow. This process is enhanced where there is an impeding soil layer that leads to the formation of perched water tables which cause soils to saturate and become highly conductive (Weiler et al., 2005).

[1] Hortonian runoff is named for Robert E. Horton (1875-1945); a pioneer of modern hydrology.

2.2.5. Runoff in Real Catchments

Although the distinctions between the various runoff mechanisms are useful and important, they may not be so clear cut in real catchments where runoff may be produced from a variety of mechanisms which vary between and during storms. Runoff processes may also differ compared to what would be expected. The runoff production that occurs during extreme events may not just be a variation on normal behaviour but the result of completely different processes. For example, infiltration excess processes may switch on during very intense rainfall in a catchment where runoff is normally contributed to by saturated source areas. In many cases, the catchment area can change if water flows across a drainage divide because of blockage or insufficient capacity of drainage structures. These issues are discussed further in the case studies in Book 4, Chapter 2, Section 7. Blockage issues are specifically addressed in Book 6, Chapter 6.

2.3. Baseflow

Streamflow is often divided into quickflow and baseflow. Quickflow is the characteristic rapid response of a stream to rainfall and catchment runoff while baseflow is contributed by slow release of stored water. Quickflow is often referred to as 'direct runoff' or as 'surface runoff' but, as noted above, can include subsurface stormflow. During floods, quickflow is of the greatest relevance but, particularly for modelling, baseflow must be considered where it provides a significant contribution to a flood hydrograph (Figure 4.2.3).

There are a range of processes that contribute to the conceptual baseflow hydrograph as shown in Figure 4.2.3. The initial baseflow represents the contribution from previous events; then as the hydrograph rises, baseflow can be depleted as water enters bank storage or is removed by transmission loss. Later, baseflow can increase as bank storage re-enters the stream, or through other processes such as interflow and discharge from groundwater (Laurenson, 1975).

Generally, quickflow will be explicitly modelled, by for example, a runoff-routing model, and then baseflow must be added to produce a flood hydrograph and unbiased estimate of the peak flow. Baseflow provides a significant contribution to peak flows in around 70% of Australian catchments (refer to Book 5, Chapter 4).

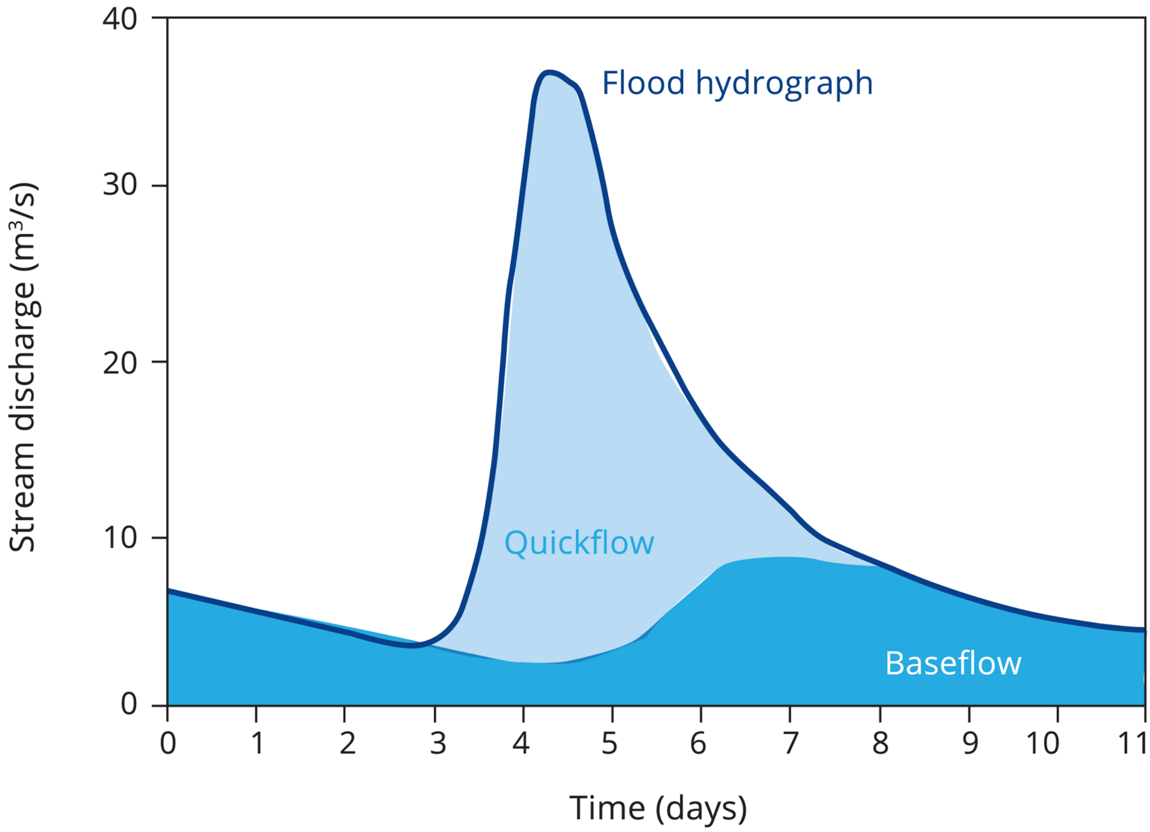

Figure 4.2.3. Observed Hydrograph - Sum of the Baseflow Hydrograph and the Quickflow

2.4. Losses

In flood hydrology, losses refer to any rainfall that is not converted to quickflow. The amount of loss is subtracted from storm rainfall to leave the “rainfall excess”, that is, quickflow is produced by the rainfall excess on the catchment. Some of the water accounted for in losses is evaporated, perhaps after being intercepted by vegetation or held in surface depressions. Some losses are infiltrated rainfall that may contribute baseflow to the stream.

Losses can be estimated for historic events. Where there are measurements of the volume of runoff, catchment area and rainfall depth, losses can be calculated as the difference between the volume of rainfall and the volume of the quickflow hydrograph (the flood hydrograph with the baseflow removed). This approach was used to estimate losses for a range of catchments as discussed in Book 5 and in earlier work on losses e.g. Hill et al. (1998).

Losses must also be predicted as part of flood forecasting and design values for losses are required as part of design flood estimation. A variety of loss models have been developed as discussed in Book 4, Chapter 2, Section 6 and in Book 5, Chapter 3, Section 2.

2.5. Flow Routing

During a flood, rainfall is converted to runoff and is transferred through a network of flow paths to the catchment outlet. These flow paths include overland flow on hill slopes, down tributaries, across floodplains, through natural and artificial storages and along main streams. Flow routing is the mathematical description of flow processes that model the attenuation and translation of hydrographs as water moves through this network. A variety of flood routing approaches are described in Book 5.

2.6. Conceptualising Processes in Models

The physical processes related to losses, runoff production, baseflow and routing need to be conceptualised and made mathematically explicit if they are to be used in modelling. This conceptualisation can vary in complexity as a function of the scales used for space and time and the representation of the underlying physics (Haan et al., 1982; Pilgrim and Cordery, 1993; Abbott et al., 1986; Beven, 2002; Beven, 2011; McDonnell, 2013; Wagener, 2003).

In general, the choice of model should depend on the amount of data that is available (Figure 4.2.4). Models that are too simple are not able to exploit the available data, while models that are too complex may suffer from 'over fitting' and have poor predictive ability. The enduring popularity of reasonably simple hydrologic models, such as RORB, is because they have been found to be of a complexity that matches the reasonably limited data that is available for most catchments.

This section briefly reviews the conceptualisation of hydrologic processes leading to floods and refers to other sections where more detail is available.

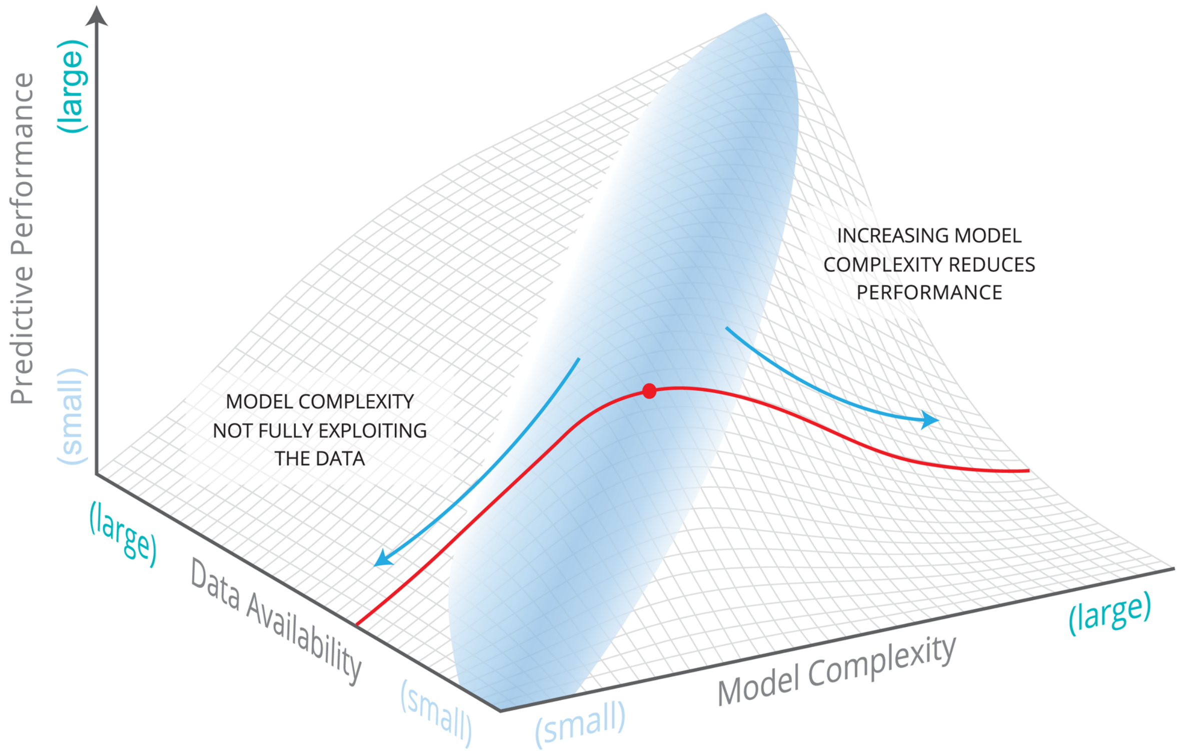

Figure 4.2.4. Conceptual Relationship between Data Availability, Model Complexity and Predictive Performance (Grayson and Blöschl, 2000)

2.6.1. Runoff Production

Models of runoff production usually require rainfall as an input, which is then allocated to surface runoff and possibly infiltration and evaporation. Rigorous approaches to modelling infiltration are available such as those based on the Richards Equation or the Green and Ampt approach (Mein and Larson, 1973; Dingman, 2002). Evaporation can be modelled as a function of meteorological drivers, soil properties and moisture content (Soil Vegetation Atmosphere Transfer (SVAT) models) (Dolman et al, 2001).

For flood modelling the physics of infiltration or evaporation, are seldom modelled explicitly, instead design or observed rainfall is converted to ‘rainfall excess’ by subtracting losses ie. the portion of rainfall that does not become direct runoff.

2.6.2. Losses

The loss models used in flood modelling are often simple, based on two parameters, one to characterise the Initial Loss (IL) (the water required to wet up the catchment) and one to characterise the Continuing Loss (CL). The output of these models is the rainfall excess that is then used to generate a direct flow hydrograph. Loss models can be standalone, ie. the rainfall excess can be calculated separately, or integrated within acatchment modelling system.

The current recommendation in ARR (Book 5, Chapter 3, Section 2) is that the IL/CL model is the most suitable for design flood estimation for both rural and urban catchments. This model uses a constant value of initial loss and constant value of continuing loss for a flood event.

For urban catchments, ARR (Book 5, Chapter 3, Section 5) provides IL and CL values for three hydrologically distinct surfaces:

- Effective Impervious Areas (impervious areas that are connected to streams by hydraulically efficient drainage);

- Pervious Areas - recommended loss values are the same as those for rural areas; and

- Indirectly Connected Areas (a combination of indirectly connected impervious and pervious areas). Recommended loss values are between those recommended for pervious and effective impervious areas.

Where losses must be estimated for flood forecasting, continuous simulation or other design problems, more complex loss model may be appropriate. Potential candidate models are discussed in Book 5, Chapter 3, Section 2.

2.6.3. Baseflow

For flood modelling, important aspects of baseflow that must be addressed are:

1. The removal of baseflow from measured hydrographs of historic flood events so that the quickflow hydrograph can be determined; and

2. The addition of a baseflow hydrograph to modelled direct flow so the total flood hydrograph, and particularly flood peak, can be correctly estimated.

Features of the baseflow hydrograph and the key processes are discussed in Book 5, Chapter 4.

When determining a design baseflow hydrograph, of particular relevance is the baseflow under the hydrograph peak as this provides a direct contribution to the maximum flood flow for an event. Procedures to estimate baseflow characteristics for design flood estimation are provided in Book 5, Chapter 4.

2.6.4. Routing

The purpose of flow routing in models is to provide a calculated estimate of the hydrograph at the downstream end of a reach given a hydrograph at the upstream end. This section briefly reviews catchment processes that are represented by routing methods in flood models. For further information on these methods, as discussed in Book 5, Chapter 5.

At any point in a stream, at a particular time during a flood event, the water flowing past will be contributed by a variety of pathways and processes that all come together to make up the flow at that instant. If we traced each drop of water within the flow, all would have originated as rainfall but have been on a variety of journeys through the catchment and travelled at different speeds: one drop of streamflow may have started as rainfall on the water surface a short distance upstream, another may have come from rain falling on saturated soil beside the river bank; yet another may have originated from a previous storm event and travelled to the stream via groundwater.

Streamflow derived from rainfall, passes through various storages. Groundwater represents long-term storage. There is also temporary storage, lasting as long as a flood event, consisting of water in transit in each element of the drainage system including water in the main stream, tributaries, hill slopes and overland flow. Water can be temporarily stored on floodplains and in retarding basins. There is also riverbank storage, water wetting up the bank profile at the start of an event and later flowing back into the stream as the water level drops.

This process description suggests routing models would need to be highly complex to represent the large number of pathways, flow speeds, and storage characteristics. However, surprisingly, simple mathematical approaches can be used to represent the movement of water along the different catchment pathways. Catchment response is usually highly damped so that short-term fluctuations in rainfall have little influence on the streamflow hydrograph and individual pathways do not need to be explicitly modelled. Instead, the dominant effect of routing is attenuation and translation which can be well represented by average response over longer time periods.

Routing of flows in a catchment may be achieved using hydrologic or hydraulic methods, and the various approaches to this are discussed in Book 5, Chapter 5. The simplest representation of routing in models is hydrologic routing which combines continuity with a relationship between storage and flow. With this approach, flow paths in a catchment are divided into a series of elements, where the volume of storage at any time is related to the discharge in each element. Differences between rural and urban streams may be represented by parameters which control the amount of water that is stored temporarily for a given flow rate. Hydrologic routing methods cannot easily accommodate backwater effects, and thus they are not well suited to situations which are influenced by tides and storm surges, or reaches in which waves propagate upstream due to the effects of large tributary inflows and waterway constrictions.

Hydraulic routing provides an increase in complexity and a reduction in the requirements for simplifying assumptions. Unsteady modelling of flows in two dimensions can be undertaken by solving the depth-averaged equations that describe the conservation of mass and momentum. These two dimesnaional (2D) models are described in more detail in Book 6, Chapter 4, Section 5 along with one dimensional (1D) unsteady models and coupled 1D/2D approaches. The limitations and appropriate use of these procedures, and others, are described in detail in Book 6.

It is possible to combine hydrologic and hydraulic routing. Hydrologic models have a short run time which facilitates the use of Monte Carlo approaches while hydraulic models are better able to represent complex routing situations. If a hydraulic model can be used to establish a storage discharge relationship, then this can be included in the hydrologic model which can then be run multiple times as part of an ensemble or Monte Carlo analysis. The use of 1D hydraulic models with short run time coupled with hydrologic models for Monte Carlo modelling is also possible.

2.6.5. Spatial Representation of Hydrological Processes

The sections above have outlined the conceptual representation of flood processes in models. Another key issue is how processes are represented spatially. In order of increasing complexity, models may be described as being lumped, semi-distributed, or distributed (Figure 4.2.5).

Lumped models (left panel of Figure 4.2.5) treat a drainage area as a single unit and use catchment averaged values of inputs and parameters. For example, spatially averaged rainfall is used as the main driver with single average values for initial and continuing loss. Simple routing approaches are used perhaps based on the passage of a hydrograph through a single storage or separate storages for surface water and groundwater. Lumped models are less common in design flood estimation or flood forecasting.

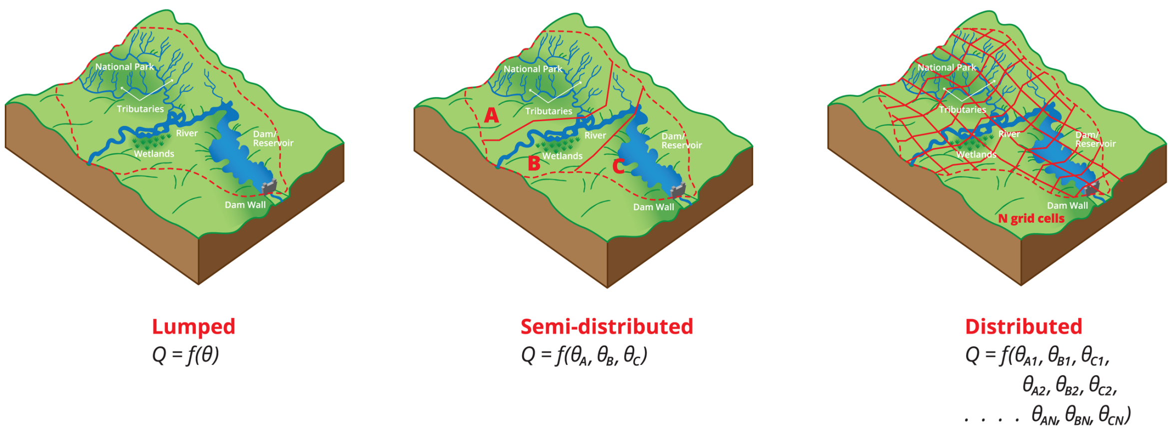

Figure 4.2.5. Spatial Representation of Physical Processes in Hydrologic Models

Semi-distributed models (middle panel of Figure 4.2.5) consider catchments as a number of reasonably large sub-areas. The spatial distribution of catchment rainfall is represented by the rainfall depth on each sub-catchment and losses and routing parameters can vary by sub-area. This approach is commonly used in design flood estimation to represents areal variations in rainfall and losses, and the effects of varying flow distance to the catchment outlet. Semi-distributed approaches can be used to create groups of hydrologic processes that are modelled in a consistent way. For example, the routing of flow down hill slopes can be modelled separately from flow routing in channels. Model setup then requires the explicit identification of hill slopes and channels that are to be modelled. The modelling equations, inputs and parameters for these areas must be provided. This group of models is discussed in detail in Book 5, Chapter 6, Section 4.

Distributed models (right panel of Figure 4.2.5) use a more spatially explicit approach, usually based on a grid that may be of a consistent size and shape across a study area or may be varied adaptively. Distributed models require inputs and parameters for each grid cell; the advantage is that results can then be produced for each grid cell. For example, two dimensional unsteady hydraulic routing approaches are commonly applied to grids to create spatially detailed information on flow depths, velocities and flood hazard in rural and urban areas.

Current approaches to design flood modelling are commonly based on a semi-distributed hydrologic model of an upper catchment area providing inputs to a distributed hydraulic model that generates outputs suitable for spatial flood mapping. The hydrologic model uses a semi-distributed approach to deal with losses and runoff generation. Hydrologic routing is used for flow down hill slopes and the upper reaches of the stream channel system. Hydraulic routing characterises flow both within channels and overbank areas where detailed information on depths and extents are required.

An alternative to combining a semi-distributed hydrologic model with a distributed hydraulic model is ‘direct rainfall’ or ‘rainfall on grid’ models. These types of models use a distributed approach to both hydrology and hydraulics by gridding an entire catchment and simulating the runoff-routing process for each grid cell. Rain falling on a grid cell is converted to runoff, after allowing for losses, and this is added to any existing flow and hydraulically routed downstream using an unsteady 2D approach. Some information on these models is provided in Book 6, Chapter 4, Section 7 with additional detail.

2.7. Examples

Three case studies are provided that outline flood runoff processes in:

- A tropical catchment (South Creek, North Queensland);
- A temperate catchment (Tarrawarra, Victoria); and
- Urban areas.

2.7.1. South Creek - North Queensland

The South Creek catchment provides a surprising example of flood runoff processes in a tropical environment with steep slopes and soils with high infiltration capacity. South Creek is 6 km east of Babinda, between Townsville and Cairns in north-east Queensland (17.35S, 145.98E) and has been well studied to determine key hydrological processes. The climate is tropical with high average annual rainfall compared to other regions of Australia. Cyclones produce rainfall intensities amongst the highest in Australia and daily rainfalls in excess of 250 mm have been reported. The catchment area is 25.7 ha with steep slopes (mean catchment slope 34%). The average saturated hydraulic conductivity of the surface soils is very high, mean value 1350 mm/hour which is higher than the rainfall intensity during the most extreme storms (the 1% Annual Exceedance Probability, 5 minute rainfall intensity is about 300 mm/hr). Saturated hydraulic conductivity decreases rapidly with depth to about 13 mm/hr below 0.2 m (Bonell et al., 1979).

At the time the South Creek catchment was instrumented, it was expected that there would be little or no overland flow. The steep, well drained and permeable slopes, along with high annual rainfall (> 4000 mm), and restricted layer at shallow depth, was expected to result in the upper layers of the soil profile becoming saturated, suggesting ideal conditions for lateral subsurface stormflow. However, this was found not to be the case.

Measurements showed that overland flow was the dominant runoff process. For example, during storms in January and March 1976, over 90% of runoff was produced by overland flow. Although rain infiltrated into the soils, the restricting layer at 200 mm depth led to a perched water table and caused saturation at the surface. Exfiltration and further rainfall landing on saturated areas, which covered most of the catchment, led to overland flow (Bonell and Gilmour, 1978).

The dominance of overland flow has implications for modelling of South Creek and similar catchments. The routing approach must be suitable for rapid flow down steep slopes that results in short lag times between rainfall and streamflow (Bonell et al., 1979). A constant continuing loss model may be suitable although this would need to be tested.

2.7.2. Tarrawarra – Southern Victoria

There has been extensive collection of hydrologic data at a small catchment (10.5 ha) at Tarrawarra 50 km ENE of Melbourne (37.66S, 145.42E), which demonstrates runoff processes in this agricultural environment (Western and Grayson, 1998). The climate is temperate with annual rainfall 820 mm and annual potential evaporation 830 mm. Evaporation exceeds rainfall in summer and surface soils dry out and crack.

During dry periods, runoff has not been observed, even during summer storms with rainfall intensities up to 50 mm/hr. When the catchment is dry, there may be local areas where rainfall intensity exceeds infiltration capacity, but any runoff that is produced enters the soil by running down surface cracks or infiltrating further down-slope and never reaches the catchment outlet.

Runoff only occurs after the catchment wets up, cracks close, and a zone of saturated soil provides a link to the catchment outlet. During wet periods, the soils at the bottom of swales saturate creating variable source areas that expand with additional rainfall. For example, during the storms of 29 and 30 July 1996 approximately half of the rainfall was converted to runoff (Western and Grayson, 2000).

The runoff production processes at Tarrawarra have implications for modelling of this type of catchment. For runoff-routing models, spatially explicit soil moisture accounting will be important as the water content of soils has a strong influence on losses (Western and Grayson, 2000). For event models, seasonal estimates of losses may be necessary. A proportional loss model may be more appropriate than one that relies on constant continuing loss.

2.7.3. Runoff from Urban Areas

Flood runoff from urban areas is larger than from rural catchments both because of catchment process and because of efficient drainage.

In an urban catchment, runoff is produced from impervious surfaces. On these surfaces: interception loss are low, because there is little vegetation; depression storage is small because the surfaces are smooth, and there is low infiltration. This means that even small amounts of rainfall will produce runoff.

In the analysis of events in urban areas, a significant feature is the small values of initial loss. Boyd et al. (1993) analysed 763 events in urban areas. For most of these, the initial loss was less than 1 mm. The average initial loss weighted by the number of events was 0.62 mm. Considering initial loss on individual catchments, information summarised in

Table 5.3.5 shows that 70% of catchments have an initial loss of 1 mm or less (refer also Book 5, Chapter 3, Section 5).

It is possible to compare the initial loss on rural and urbanised catchments. Data for Australian catchments is summarised in the Appendix to Book 5 (Book 5, Chapter 3, Section 8). A density plot of this data shows the substantially lower initial loss for urban catchments and the concentration near 0 mm. For rural catchments, the mean initial loss across all catchments is 32 mm but the high standard deviation (16.8 mm) means the density is spread across a wide range of values.

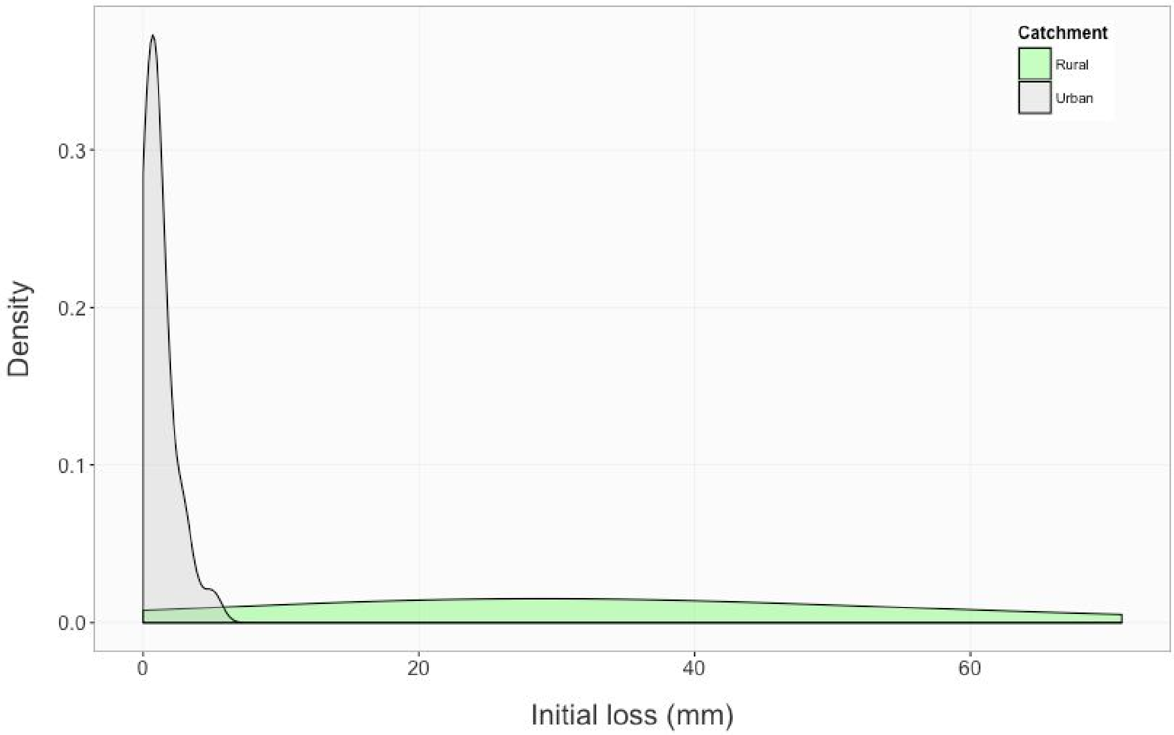

Figure 4.2.6. Comparison of Initial Loss in Urban and Rural Catchments

During larger events in urban areas, pervious surfaces also produce runoff either through infiltration excess or saturation excess processes. Many pervious surfaces in urban areas are compacted because they are walked or driven on, decreasing infiltration capacity and increasing the proportion of rainfall running off.

Along with these catchment processes, the piped drainage system in urban areas efficiently delivers water to streams. Piped drainage represents an extension of the drainage network so that even areas distant from the original natural waterways contribute flow to those waterways. In highly urbanised catchments every impervious surface will be drained to the stream.

In addition to catchment changes, the modification to urban streams also changes the transfer of flood flows. Modified urban streams have less attenuation, transmission losses are reduced and water travels more quickly. The results is a substantial increase in magnitude and frequency of flooding. Further details are provided in Book 9.

Modelling urban hydrology can be challenging because of the variety of different surface types and variation in connections between surfaces and drains. There are parallel flow paths with different routing characteristics. Some water will pass through the piped system while some will flow overland. Water can surcharge out of pipes or enter pipes at various locations in the catchment. Flow behaviour, and even catchment area, depends on flood magnitude. Modelling approaches for urban areas are discussed in Book 9.

2.8. References

ASCE (American Society of Civil Engineers) (1975), Aspects of hydrological effects of urbanisation. ASCE Task Committee on the effects of urbanisation on low flow, total runoff, infiltration, and groundwater recharge of the Committee on surface water hydrology of the Hydraulics Division. Journal of the Hydraulics Division, (5), 449-468.

Abbott, M.B., Bathurst, J.C., Cunge, J.A., O'Connell P.E. and Rasmussen, J. (1986), An introduction to the European hydrological system- Systeme Hydrologique Europeen, SHE, 1: history and philosophy of a physically-based, distributed modelling system. Journal of Hydrology, volume 87(1-2), pp.45-59.

Beven, K.J. (2002), Towards an alternative blueprint for a physically based digitally simulated hydrologic response modelling system, Hydrological Processes, 16: 189-206.

Beven, K.J. (2011), Rainfall-Runoff Modelling, Wiley (2nd Edition).

Bonell, M. and Gilmour, D.A. (1978), The development of overland flow in a tropical rainforest catchment. Journal of Hydrology, 39: 365-382.

Bonell, M., Gilmour, D.A. and Sinclair, D.F. (1979), A statistical method for modelling the fate of rainfall in a tropical rainforest catchment. Journal of Hydrology, 42: 251-267.

Boyd, M.J., Bufill, M.C. and Knee, R.M. (1993), Pervious and impervious runoff in urban catchments. Hydrological Sciences Journal, 38(6), 463-478.

Cordery, I. (1976), Some effects of urbanisation on streams. Civil Engineering Transactions, The Institution of Engineers, Australia, CE18(1), 7-11.

Cordery, I. and Pilgrim, D.H. (1983), On the lack of dependence of losses from flood runoff on soil and cover characteristics. Hydrology of Humid and Tropical Regions with Particular Reference to the Hydrological Effects of Agriculture and Forestry Practice (Proceedings of the Hamburg Symposium). Hamburg, IAHS Publication No.140.

Dingman, S.L. (2002), Physical hydrology. Prentice Hall.

Dolman, A.J., Hall, A.J., Kavvas, M.L., Oki, T. and Pomeroy, J.W. (2001), Soil-Vegetation-Atmosphere transfer schemes and large-scale hydrologic models. Proceedings on an international symposium held during the sixth Scientific Assembly of the International Association of Hydrological Sciences (IAHS) at Maastricht, The Netherlands 18-27 July 2001. IAHS Publication No.270.

Dunne, T. and Black, R.D. (1970), Partial area contributions to storm runoff in a small new England watershed. Water Resources Research, 6(5), 1296-1311.

Espey, W.H. and Winslow, D.E. (1974), Urban flood frequency characteristics. Journal of the Hydraulics Division. American Society of Civil Engineers, 100(HY2), 279-293.

Ferguson, B.K. and Suckling, P.W. (1990), Changing rainfall-runoff relationships in the urbanizing Peachtree Creek Watershed, Atlanta, Georgia. Water Resources Bulletin, 26(2), 313-322.

Grayson, R.B. and Blöschl, G. (2000), Spatial Patterns in Catchment Hydrology: Observations and Modelling. Cambridge University Press, p: 404

Haan, C.T., Johnson, H.P. and Brakensiek, D.L. (1982), Hydrologic modelling of small watersheds. American Society of Agricultural Engineers. St Joseph, Michigan.

Harris, E.E. and Rantz, S E. (1964), Effect of urban growth on streamflow regimen of Permanente Creek Santa Clara County, California: Hydrological effects of urban growth. Geological survey water-supply paper 1591-B. US Department of the Interior.

Hill, P.I., Mein, R.G. and Siriwardena, L. (1998), How much rainfall becomes runoff: loss modelling for flood estimation. Cooperative Research Centre for Catchment Hydrology.

Hollis, G.E. (1975), The effect of urbanisation on floods of different recurrence interval. Water Resources Research, 11: 431-435.

Laurenson, E.M (1975), Streamflow in catchment modelling. In: Chapman, T. G. and Dunin, F. X. (eds) Prediction in catchment hydrology. Australian Academy of Science, pp.149-164.

Laurenson, E.M. and Pilgrim, D.H. (1963), Loss rates for Australian Catchments and their Significance Journal, Institution of Engineers, Australia, 35(1-2), 9-24.

McDonnell, J.J. (2013), Are all runoff processes the same? Hydrological Processes, 27: 4103-4111.

Mein, R.G. and Goyen, A.G. (1988), Urban runoff. Civil Engineering Transactions, 30: 225-238.

Mein, R.G. and Larson, C.L. (1973), Modelling infiltration during steady rain. Water Resources Research, 9(2), 384-394.

Pilgrim, D.H. and Cordery, I. (1993), Flood runoff. In: Maidment, D.R. (ed) Handbook of Hydrology. McGraw-Hill, pp: 9.1-9.42.

Tholin, A.L. and Keifer, C.J. (1959), The hydrology of urban runoff. Journal of the Sanitary Engineering Division, 85(SA2), 47-106.

Wagener, T. (2003). Evaluation of catchment models, Hydrological Processes, 17.16:3375-3378.

Weiler, M., McDonnell, J.J., Tromp-van Meerveld, I. and Uchida, Taro. (2005), Subsurface Stormflow. In: Anderson, M.G. (ed.) Encyclopedia of Hydrological Sciences. John Wiley and Sons.

Western, A.W. and Grayson, R.B. (1998), The Tarrawarra data set: soil moisture patterns, soil characteristics and hydrological flux measurements. Water Resources Research 34(10), 2765-2768.

Western, A.W. and Grayson, R.B. (2000), Soil moisture and runoff processes at Tarrawarra. In: Grayson, R. B. and Blöschl G. (eds) Spatial patterns in catchment hydrology: observations and modelling. Cambridge University Press, pp: 209-246.

Chapter 3. Types of Simulation Approaches

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Chapter Status	Final
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3.1. Introduction

Rainfall-based models are commonly used to extrapolate flood behaviour at a particular location using information from a short period of observed data. This can be done using either event-based or continuous simulation approaches.

Event-based approaches are based on the transformation of a discrete rainfall event into a flood hydrograph using a simplified model of the physical processes involved. It requires the application of two modelling steps, namely: a *runoff production* model to convert the storm rainfall input at any point in the catchment into rainfall excess or runoff at that location, and a *hydrograph formation* model to simulate the conversion of these local runoffs into a flood hydrograph at the point of interest. The rainfall event is described by a given depth of rainfall occurring over a selected duration, where it is necessary to specify the manner in which the rainfall varies in both time and space. The input rainfall may represent a particular observed event, or else it may represent the depth of rainfall with a specific Annual Exceedance Probability (ie. a design rainfall). The former approach is most commonly used for model calibration and flood forecasting, the latter approach is used to estimate flood risk for design and planning purposes. The defining feature of such models is that they are focused on the simulation of an individual flood event, and that antecedent (and baseflow) conditions need to be specified in some explicit fashion.

In contrast, continuous simulation approaches transform a long time series of rainfall (and other climatic inputs) into a corresponding series of streamflows. Such time series may span many weeks or years, and may represent behaviour that reflects the full spectrum of flood and drought conditions. Such models comprise simplified representation of catchment processes, and most usually involve the simulation of soil moisture and its control over the partitioning of rainfall into various surface and subsurface contributions to recharge and streamflow. Once simulated, information on the frequency and magnitude of flood behaviour needs to be extracted from the resulting time series using the same methods adopted for historical streamflow data.

The relative strengths and weaknesses of these approaches are outlined in Book 1, Chapter 3. The following sections provide information on simulation approaches relevant to each approach, where guidance on their calibration and application is presented in Book 7. Event-based models may be implemented in a variety of ways, and three approaches of increasing sophistication are described in Book 4, Chapter 3, Section 2 to Book 4, Chapter 3, Section 2. The Simple Event approach is first described in Book 4, Chapter 3, Section 2, and this includes discussion of the main elements that are common to all event-based approaches. The Ensemble Event approach (Book 4, Chapter 3, Section 2) provides a simple means to accommodate variability of a selected input, and this is followed by description of Monte Carlo approaches in Book 4, Chapter 3, Section 2, which provide a rigorous treatment of the joint probabilities involved in estimation of design floods. Continuous Simulation approaches are described in Book 4, Chapter 3, Section 3, and hybrid approaches based on a mixture of event- and continuous schemes are briefly described in Book 4, Chapter 3, Section 4. The performance, strengths and limitations of the different approaches are discussed in Book 4, Chapter 3, Section 5 and Book 4, Chapter 3, Section 6, and finally, the elements of a worked example are presented in Book 4, Chapter 3, Section 7.

3.2. Event-Based Approaches

3.2.1. General Concepts

Event-based approaches represent traditional practice in Australia and most overseas countries for derivation of design floods from design rainfalls. Typical hydrologic inputs to event-based models include:

- A *design storm* of preselected AEP and duration: historically it has been most common to only consider the most intense parts of complete storms ("design burst"), where the average intensity of the burst is determined from rainfall Intensity Frequency Duration (IFD) data (Book 2, Chapter 2). This information is generally available as a point rainfall intensity, and it is necessary to apply an Areal Reduction Factor (Book 2, Chapter 4) to correctly represent the areal average rainfall intensity over a catchment;
- *Temporal patterns* to distribute the design rainfall over the duration of the event, and this can include additional rainfalls before the start (and after the end) of the burst to represent complete storms (Book 2, Chapter 5);
- *Spatial patterns* to represent rainfall variation over a catchment that occurs as the result of factors such as catchment topography and storm movement (Book 2, Chapter 4); and
- *Loss parameters* that represent soil moisture conditions in the catchment antecedent to the event and the capacity of the soil to absorb rainfall during the event (Book 5, Chapter 5).

A range of event-based models are available to convert rainfalls into a flood hydrograph, though in generally these models provide highly simplified representations of the key processes relevant to flood generation:

- *A loss model* is used to estimate the portion of rainfall that is absorbed by the catchment and the portion that appears as direct runoff (Book 5, Chapter 3). This loss is typically attributed to a range of processes, including: interception by vegetation, infiltration into the soil, retention on the surface (depression storage), and transmission loss through the stream bed and banks; and
- *A hydrograph formation model or hydrologic routing model* (usually based on runoff-routing concepts, as discussed in Book 5, Chapter 6) is used to transform the patterns of rainfall excess into a design flood hydrograph. This flood hydrograph may include a baseflow component which initially represents the delayed contribution from previous rainfall events, and in the latter stages of the event may represent the contribution from earlier losses.

The most commonly applied event-based approach is the Design Event approach which assumes that there is a *critical rainfall duration* that produces the design flood for a given catchment. This critical duration depends on the interplay of catchment and rainfall characteristics; it is not known *a priori* but is usually determined by trialling a number of rainfall durations and then selecting the one that produces the highest flood peak (or volume) for the specific design situation.

An important consideration in the application of this approach is that the inputs defining the Design Event should be selected to be probability neutral. This involves selecting model inputs and parameter values such that the 1 in X AEP design rainfalls are converted to the corresponding 1 in X AEP floods. The task of defining a typical combination of flood producing factors for application in the ‘Design Event’ approach is made particularly difficult by the fact that flood response to rainfall is generally non-linear and can be highly non-linear. This means that average conditions of rainfall or loss are unlikely to produce average flood conditions. The probability neutrality of inputs can only be tested if independent flood estimates are available for comparison; for more extreme events, the adopted values of probability neutral inputs must be conditioned by physical and theoretical reasoning.

The following guidance presents three approaches to dealing with probability neutrality, namely:

- *Simple Event*, where all hydrologic inputs are represented as single probability neutral estimates from the central range of their distribution;
- *Ensemble Event,* where the dominant factor influencing the transformation is selected from a range of values representing the expected range of behaviour, and all other inputs are treated as fixed; and
- *Monte Carlo Event,* where all key factors influencing the transformation are stochastically sampled from probability distributions or ensembles, preserving any significant correlations between the factors, and probability neutrality is assured (for the given set of inputs) by undertaking statistical analysis of the outputs.

The key differences between these approaches is illustrated in Figure 4.3.1.Book 4, Chapter 3, Section 2 to Book 4, Chapter 3, Section 2 describe each of these procedures in turn, though it is worth noting here the essential similarities between the three methods as shown in Figure 4.3.1. It is seen that these three methods use the same source of design rainfalls and the same conceptual model to convert rainfall into a flood hydrograph. The process involved in calibrating a conceptual model to historic events is common to all three approaches, they differ only in how selected inputs are treated when deriving design floods.

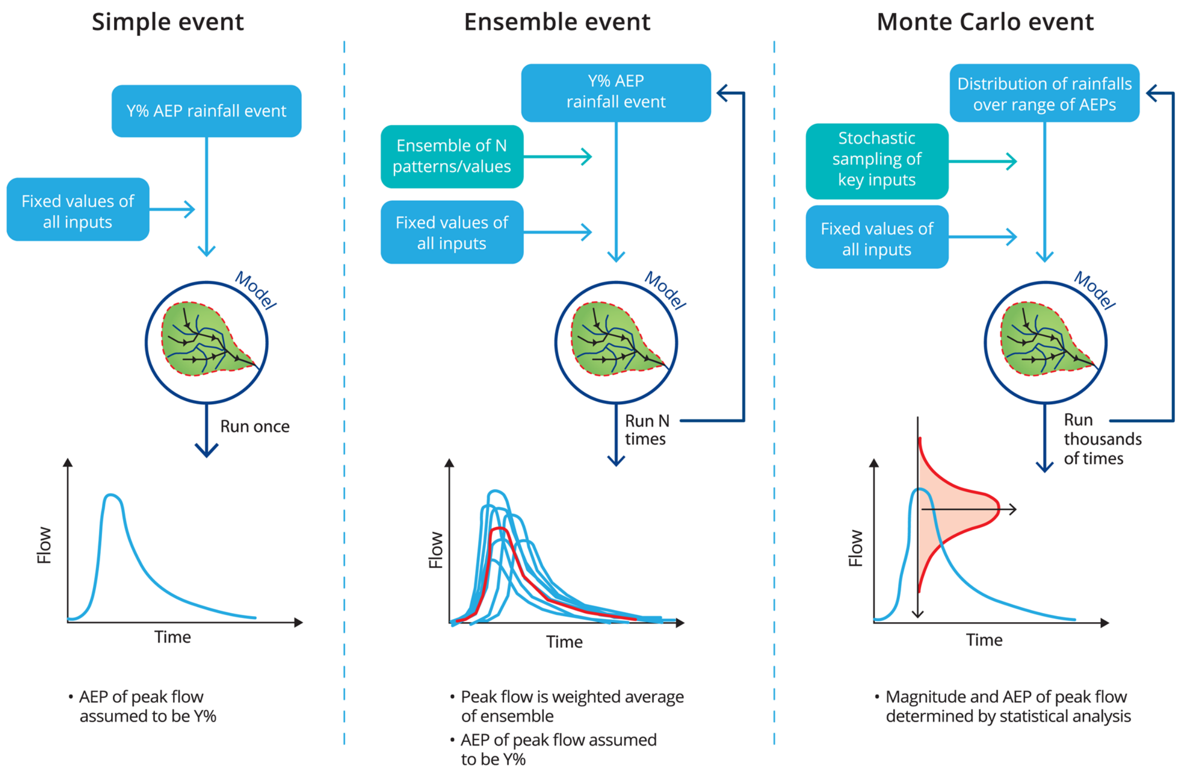

Figure 4.3.1. Elements of Three Different Approaches to Flooding

3.2.2. Simple Event

As shown in Figure 4.3.1, the first step in the Simple Event method is to estimate the average intensity or depth of rainfall corresponding to a given AEP for a selected duration using Intensity Frequency Duration (IFD) data, as provided in Book 2, Chapter 2. The next step is to select representative values of other factors that influence the transformation of rainfall to flood hydrograph. At a minimum, this involves selecting representative temporal and spatial patterns of rainfall, and selecting appropriate loss parameters.

Representative temporal patterns of rainfall may be obtained by applying the Average Variability Method to a sample of historic patterns (Pilgrim et al., 1969; Pilgrim and Cordery, 1975). The intent of this method is to derive a single temporal pattern which is representative of the average variability of intense rainfall relevant to the selected storm duration and severity. Their use is based on the assumption that such patterns should minimise the introduction of joint probabilities into the design flood model and aid in estimation of a flood with the same frequency as the design rainfall. However, there is good evidence that patterns of average variability do not ensure probability neutrality (e.g. Sih et al. (2008), and Green et al. (2003)), and it is possible that adoption of historical patterns selected from within the range of observed variability are as efficacious as synthetic ones derived using the Average Variability Method. Temporal patterns based on the Average Variability Method have been developed for point rainfalls up to the 1 in 500 AEP (Pilgrim (1987) Volume 2) and for areal Probable Maximum Precipitation estimates (Nathan, 1992; Green et al., 2003).

Spatial patterns of rainfall generally have a lower influence on flood characteristics than temporal patterns, and consequently simpler approaches are used to accommodate the joint probabilities involved. For most practical situations it is assumed sufficient to adopt a fixed non-uniform pattern that reflects the systematic variation arising from topographic influences (Book 2, Chapter 4).

For estimating losses, various types of models ranging from a simple loss model to complex conceptual runoff-routing models are available (Hoang et al., 1999; Hill et al., 2012). Loss models most suited for design purposes generally involve specification of a parameter (such as initial loss) that is related to soil moisture conditions in the catchment prior to the onset of the storm. They also generally involve specification of a loss term related to the infiltration of a proportion of storm rainfall during the event (e.g. continuing loss or proportional loss). The most comprehensive analyses of design loss values available to date have been undertaken by Kuczera et al. (2006) and Newton and Walton (2000), and guidance on suitable loss values to adopt is provided in Book 5, Chapter 5. The selected loss values can have a large influence on the resulting flood characteristic, and the adoption of regional estimates does not guarantee unbiased estimates of the resulting floods; for this reason it is also desirable to reconcile design values with independent flood frequency estimates where possible (as discussed in Book 5, Chapter 5).

The direct runoff simulated by the loss model is then routed through the catchment to generate the design flood hydrograph. The hydrograph corresponding to the rainfall burst duration that results in the highest peak (the critical rainfall duration) is taken as the design flood hydrograph, and it is assumed to have the same Annual Exceedance Probability as its causative rainfall. It needs to be stressed that probability neutrality is an untested assumption with the simple event approach, and without reconciliation with flood frequency estimates using at-site or transposed gauged maxima, there is no way of determining how the selected inputs may have biased the outcome.

In summary, the only probabilistic variable considered with the Simple Event approach is average rainfall intensity or depth, while other inputs (e.g. losses, rainfall temporal and spatial patterns) are represented by fixed values drawn from the central tendency of their distribution (Rahman et al., 1998; Nathan et al., 2002; Rahman et al., 2002a; Kuczera et al., 2006; Nathan et al., 2003).

3.2.3. Ensemble Event

The Ensemble Event approach is essentially an intermediate step between a Simple Event approach and Monte Carlo Event simulation. In its simplest implementation, a fixed factor with large influence on flood magnitude is replaced by a sample of values (an “ensemble”); each of these values is then input to the flood event model to derive a set of flood hydrographs. The magnitude of the design flood is then estimated from the weighted average of the hydrographs, where the weighting applied to each result reflects the relative likelihood of the selected input occurring. If a sample of observed temporal patterns is used instead of a single pattern of average variability, then studies have shown (Sih et al., 2008; Ling et al., 2015) that a simple arithmetic average based on a sample of 10 to 20 patterns provides a reasonably unbiased estimate of the design flood. The rationale for this approach is that each of the patterns selected for the ensemble is equally likely.

In concept the approach could be extended to take account of factors that are non-uniformly distributed, though here it would be necessary to carefully weight the outcome by the relative likelihood of the different values selected, or else select the input values in a way that reflects the form of their distribution. For example, if a sample of ten initial loss values were selected, then it would be necessary to weight each result by the probability of each loss value occurring, which could be determined (for example) from the cumulative distribution of losses presented in Book 5, Chapter 5; alternatively, the distribution of losses could be divided into ten equally likely exceedance percentile ranges, and the results then be given equal weighting.

It is expected that the approach is most suited to the consideration of temporal patterns, as suitable ensemble sets of patterns are readily available (as described in Book 2, Chapter 5). Flood magnitudes are generally very sensitive to temporal patterns and thus the ensemble approach provides a straightforward, if somewhat tedious, means of avoiding the introduction of bias due to this source of variability. Extending the ensemble method to consider other inputs, jointly or otherwise, would appear to introduce additional problems which are probably most easily handled by Monte Carlo approaches.

3.2.4. Monte Carlo Event

Monte Carlo methods provide a framework for simulating the natural variability in the key processes that influence flood runoff: all important flood producing factors are treated as stochastic variables, and the less important ones are fixed. The primary advantage of the method is that it allows the exceedance probability of the flood characteristic to be determined without bias (subject to the representativeness of the selected inputs).

In the most general Monte Carlo simulation approach for design flood estimation, *rainfall events of different duration* are sampled stochastically from their distribution. The simulated design floods are then weighted in accordance with the observed frequency of occurrence of rainfall events of different durations that produced them. This avoids any positive bias of estimated flood probabilities which may be associated with the application of the critical rainfall duration concept (Weinmann et al., 2000; Weinmann et al., 2002; Rahman et al., 2002b). The application of this generalised approach relies on the derivation of new design data for rainfall events that are consistent with a new probabilistic definition of storm 'cores' or complete storms (Hoang et al., 1999). Such design rainfall data is currently not available, thus limiting the application of the generalised approach. To obviate the need for this, Nathan et al. (2002) and Nathan et al. (2003) adapted the approach to separately consider different rainfall durations; the resulting peak flows are then enveloped to select the critical event duration, consistent with the 'critical rainfall duration' concept used in traditional design flood estimation practice. This is the approach further described herein. Whilst adherence to the 'critical duration' concept could possibly introduce systematic bias into the results, it has the advantage of ensuring consistency with existing design approaches and allows much of the currently available design data to be readily used.

Undertaking a Monte Carlo simulation requires three sets of key decisions, followed by a simulation step that involves construction of the derived flood frequency curve. The overall steps involved are as follows:

i. *Select an Appropriate Flood Event Simulation Model* - The criteria for selecting an appropriate model are similar to those used with the traditional Design Event approach and are described in Book 5. The selected model should be able to be run in batch mode with pre-prepared input files or be called from the Monte Carlo simulation application. Models with fast execution speeds are well suited to Monte Carlo simulation; complex models with slow run-times can still be utilised, though generally they need to be invoked within a stratified sampling scheme (Book 4, Chapter 4, Section 3) to ensure that the simulations times are within practical constraints.

ii. *Identify the Model Inputs and Parameters to be Stochastically Generated* - The stochastic representation of model inputs should focus on those inputs and parameters which are characterised by a high degree of natural variability and a non-linear flood response. Examples include rainfall temporal pattern, initial loss and reservoir storage content at the start of a storm event. If the assessment indicates limited variability and essentially linear system response, then there may be little to be gained from extending the Monte Carlo simulation approach to include such additional inputs or parameters.

iii. *Define the Variation of Inputs/Parameters by Appropriate Distributions and Correlations* - The considerations and methods applicable to joint probability aspects are described in Book 4, Chapter 4. The distributions used to generate the stochastic inputs can be defined by the use of specific theoretical probability distributions or else an empirical, non-parametric approach can be adopted. Schaefer and Barker (2002) and Schaefer and Barker (2004) adopts a strongly parametric approach to sampling a wide range of storm and catchment processes, Rahman et al. (2002b) and Rahman et al. (2002a) provides examples in which both losses and temporal patterns are defined using a Beta distribution. (Nathan et al., 2003) and (Nathan and Weinmann, 2004) adopt a more empirical approach that is more closely aligned to the nature of design information used in the traditional Design Event method. If any of the stochastic inputs exhibit significant correlations, their correlation structure needs to be defined, and the correlations included in the sampling scheme.

iv. *Undertake Monte Carlo Simulation* - The design inputs and parameters exhibiting significant variability are sampled in turn from their distributions allowing for significant correlations, and the resulting combination of inputs and parameters is then used in a simulation model run. Only those inputs that have a significant influence on the results need to be stochastically sampled, and other inputs can be treated as fixed (usually average or median) values. For Monte Carlo simulation involving several stochastic variables, many thousands of simulations are required to adequately sample the inherent variability in the system, and thus for most practical problems some thought is required to minimise disc storage space and simulation times.

v. *Construct the Derived Flood Frequency Curve* - Once the required number of runs has been undertaken, it is necessary to analyse the results to derive the exceedance probabilities of different flood magnitudes. Where very simple models are used or the probabilities of interest are not extreme – more frequent than, say, 1 in 100 Annual Exceedance Probabilities (AEP) – the simulation results can be analysed directly using frequency analysis (as described in Book 3, Chapter 2). Alternatively, in order to estimate rarer exceedance probabilities (or use more complex models with slow execution speeds) it is desirable to adopt a stratified sampling approach to derive the expected probabilities of given event magnitudes, as described in Book 4, Chapter 4.

An example flowchart for the last two steps is illustrated in Figure 4.3.2. This flowchart represents the high level procedure relevant to the consideration of the joint probabilities involved in the variation of loss parameters and temporal patterns. The starting point for this simple Monte Carlo simulation is the Step “A” in Figure 4.3.2. The loss and temporal patterns are then sampled and combined with fixed values of other inputs for simulation using a flood event model. Once many thousands of combinations of rainfall depth, losses and temporal patterns have been undertaken, the resulting flood maxima are analysed to derive unbiased estimates of flood risk (represented by Step “B”, Figure 4.3.2). Suitable sampling schemes and analyses relevant to these steps are described in Book 7, Chapter 7, where additional variables (such as reservoir level or rainfall spatial pattern) can be included as additional sampling steps as required.

Figure 4.3.2 also depicts the relationship between Monte Carlo schemes and the other simpler event-based methods discussed above. The blue-shaded shapes represent the steps involved in the traditional Simple Event (or Design Event) approach, where the flood characteristic obtained from a single simulation using the selected inputs (Step “C”) is assumed to have the same Annual Exceedance Probability as its causative rainfall. The ensemble approach is shown as an added loop: in this example the simulation would be repeated for each available temporal pattern, and the results would be averaged (at Step “C”) to yield the flood characteristic of interest, where again it is assumed that the Annual

Exceedance Probability of the calculated flood is the same as its causative rainfall. The 2nd and 3rd last shapes represent the additional steps required to implement a Monte Carlo scheme.

It should be noted that the steps involved between points A and B in Figure 4.3.2 represent the scheme required to consider the joint probabilities associated with the variability of selected inputs. It represents the characterisation of *aleatory uncertainty*, which is the (irreducible) uncertainty associated with variability inherent in the selected inputs. However, Monte Carlo schemes can also be used to consider *epistemic uncertainty*, and the additional steps involved in this are shown by the first and last steps in Figure 4.3.2 Epistemic (or reducible) uncertainty is due to lack of knowledge, and is associated with errors in the data or the simplifications involved in representing the real world by a conceptual model. In essence, the consideration of aleatory uncertainty allows the derivation of a single (probability neutral) “best estimate” of flood risk, and consideration of epistemic uncertainty allows the characterisation of confidence limits about this best estimate. The outer (dark blue-shaded) iteration loop shows extension of approach to estimate confidence limits. Figure 4.3.2 has inner (blue-shaded) shapes that show steps involved in Simple Event approach, where dashed lines indicate additional iteration required for Ensemble Event approach.

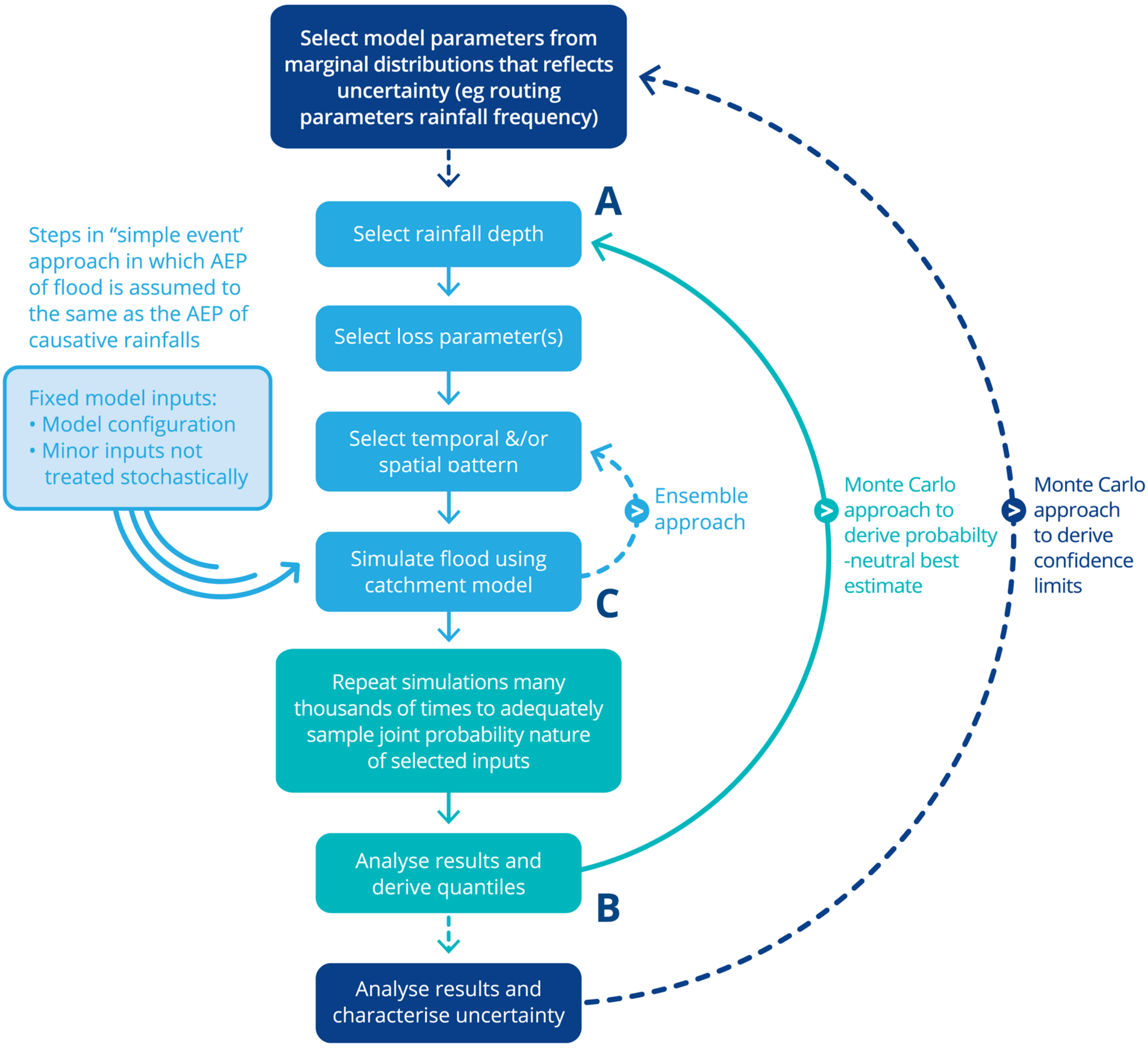

Figure 4.3.2. Simple Framework for Monte Carlo Simulation for Handling Joint Probabilities Associated with Both Losses and Temporal Patterns

In general, while the information required to characterise aleatory uncertainty can be readily obtained from the observed record, this is not the case with epistemic uncertainty. Indeed it is quite difficult to obtain information on the likely errors associated with input data or model parameterisation, and it is very difficult to characterise the uncertainty associated with model structure. Accordingly, the guidance presented here focuses on the assessment of aleatory uncertainty as it is considered that this approach can be readily understood and applied by practitioners with the appropriate skills. Thus, while it seems reasonable to regard the use of Monte Carlo procedures to accommodate hydrologic variability as "best practice" for many practical design problems, its use to derive confidence limits is expected to remain the domain of more academic specialists for the foreseeable future.

3.3. Continuous Simulation Approaches

3.3.1. General Concepts

The last few decades have seen considerable advances in computational power. This has allowed implementation of models that are more complex and that provide greater (and more elaborate) representation of the physical processes occurring in a catchment (Boughton and Droop, 2003). This has led to development of large numbers of runoff-routing models from the highly conceptualised Stanford Watershed Model (Linsley and Crawford, 1960) to more physically based models such as the Systeme Hydrologique Europeen Model (SHE; (Abbott et al., 1986)). Traditionally, rainfall based methods of estimating the design flood have predominately been event-based, while continuous simulation has been applied for yield estimation or flow forecasting. However, development of tools and methods that allow generation of long periods of synthetic rainfall data has led to increased interest in using continuous simulation for design flood estimation and the concept of using models traditionally developed for yield estimation for the estimation of design floods (Boughton and Droop, 2003).

The Continuous Simulation method of estimating the design flood is similar in intent to the event-based Monte Carlo approach discussed in Book 4, Chapter 3, Section 2. Both methods seek to adequately simulate the interactions between flood producing (rainfall and catchment characteristics) variables (Kuczera et al., 2006). Conceptually, the differences between the two methods arise in how wet and dry periods are sampled and incorporated into the process of estimating the design flood. In the event-based Monte Carlo method rrunoff-routing models are used to simulate the interactions occurring only during the storm (wet period) event. There is implicit consideration of the influence of dry periods in sampling the catchment-rainfall interactions (antecedent conditions, temporal patterns, storm durations) from exogenously derived distributions of initial conditions (Kuczera et al., 2006). The Continuous Simulation method, on the other hand, accounts for these interactions through direct simulation of the processes occurring in the catchment over an extended period (Kuczera et al., 2006; Boughton et al., 1999; Cameron et al., 1999). The Continuous Simulation method is also applicable in situations where the critical event duration extends over many weeks or months, as is the case for systems with large storage capacity but limited outflow capacity.

The Continuous Simulation method of estimating the design flood involves running a conceptual runoff-routing model for a long period of time such that all important interactions (covering the dry and wet periods) between the storm (intensity, duration, temporal pattern) and the catchment characteristics are adequately sampled to derive the flood frequency distribution. In general, pluviograph data of hourly resolution (or less) is used to drive the runoff-routing models. In most cases the period of record of pluviograph data rarely exceeds 20 years, therefore rainfall data is extended by using stochastic rainfall data generation. The runoff-routing model is calibrated using flow data, where available, and the calibrated model is then used to generate a long series of simulated flow. Finally the simulated flow is then used to extract the Annual Maximum Series and estimate the derived flood frequency curve. Important components of the Continuous Simulation approach are further discussed in the following sections:

- Stochastic Rainfall Data Generation; and
- Applications to Design Food Estimation.

3.3.2. Stochastic Rainfall Data Generation

The effectiveness of the Continuous Simulation method depends upon the availability of a sufficiently long rainfall data set to provide adequate information on extreme storm (and drought) events. In reality however, pluviograph data rarely extends beyond 50 years, and the inference of floods greater than 2% AEP is difficult (Boughton et al., 1999).

In such cases stochastic rainfall generation has been used to provide a long time series of synthetic rainfall (Boughton et al., 1999; Cameron et al., 1999; Droop and Boughton, 2002; Haberlandt and Radtke, 2013). The synthetic data set thus generated is designed to be statistically indistinguishable from observed rainfall data (Kuczera et al., 2006).

There are well established methods to generate stochastic data at a coarse time scale. However, generating fine resolution synthetic data that can reproduce the statistics of the observed rainfall series at various temporal scales (annual, monthly, daily and hourly) is challenging (Srikanthan and McMahon, 2001; Boughton and Droop, 2003; Kuczera et al., 2006). Therefore, a commonly used approach is to generate the synthetic rainfall data at a daily time step first, and then disaggregate to a sub-daily time step by using functional relationships between daily and sub-daily rainfall statistics. Boughton (1999) used the Transition Probability Matrix (TPM) model to generate thousands of years of daily rainfall data and then disaggregated the daily data to an hourly time-step using the sub-daily rainfall statistics derived from IFD curves and temporal patterns. Kuczera et al. (2006) tested the ability of the DRIP rainfall generating model Heneker et al. (2001) to reproduce observed rainfall statistics at different levels of aggregation (hourly to yearly) and found that the model was able to reproduce the observed rainfall statistics satisfactorily for the large storms.

Techniques are available for generating daily rainfalls at any site in Australia (Book 2, Chapter 7) thus the inputs required for continuous simulation models can be developed for catchments without adequate at-site rainfall data.

3.3.3. Runoff- Routing Model

Types of runoff-routing models used to simulate the flow can be varied and depend upon the complexity required to provide unbiased simulation of the hydrologic process in the catchment. For example, Boughton (1999) and Droop and Boughton (2002) used a simple lumped Australian Water Balance Model (AWBM) to simulate a long series of precipitation excess, for small to mid-sized catchments, which were then routed using an hourly hydrograph generation model. Haberlandt and Radtke (2013) used HEC-HMS (Feldman, 2000), a semi distributed rainfall-runoff model, in three medium sized catchments in Germany. Cameron et al. (1999) applied a semi-distributed conceptual runoff-routing model known as TOPMODEL (Beven et al, 1987) for design flood estimation in small sized catchments in the UK. For large catchments with large spatial heterogeneity, England (2006) recommends using a physically based distributed model to fully characterise the spatial distribution of the processes occurring in the catchment. Other commonly used continuous simulation models include SIMHYD (Chiew and McMahon, 2002), Sacramento Model (Burnash et al., 1973) and GR4H (Mathevet, 2005).

The three factors that need to be considered when selecting a continuous simulation model for flood estimation are:

1. The ability of the model to represent the physical processes occurring in the catchment (model complexity);

2. Adequate temporal resolution to simulate the embedded flood hydrographs; and

3. The amount of data and computational resources available to properly describe and calibrate the model (model parsimony).

Useful guidance on the trade-offs involved in matching model complexity with data availability is provided in (Vaze et al., 2012).

3.3.4. Model Calibration

Implementation of Continuous Simulation, and the use of synthetic data, is complicated by the need to calibrate both the rainfall data generation model and the runoff-routing model using the observed data set. Effective calibration depends upon the calibration method applied, the length and the quality of data used for calibration. Gupta and Sorooshian (1985) report that the benefit of using additional data (with similar information content) diminishes with the reciprocal of the square root of the number of data points used in the calibration. Therefore, while the length of data is an important factor, the data series should also contain a sufficient number of ‘unusual events’ (or extreme events) to enable estimation of the parameter values (Singh and Bárdossy, 2012).

The rainfall generation model is generally calibrated to storm events, as in alternating renewal models like DRIP, or to aggregation statistics (such as mean, skewness, coefficient of variation, auto correlations etc.) at various time scales (Kuczera et al., 2006). The runoff-routing models are calibrated to observed flow data, flow statistics (Boughton et al., 1999) and in some cases the flood frequency curve (Cameron et al., 1999). The alternative calibration strategies will result in different model parameter values, leading to differing representation of hydrographs and peak events.

Lack of observed data is a major problem for calibration of the rainfall generation model or the runoff-routing model. In the case of the rainfall generation model, for example, the short rainfall data sets generally available are unlikely to include extreme rainfall events caused by various rain producing mechanisms (for example cyclones vs. thunderstorms) and to sample the full range of natural variability.

3.3.5. Applications to Design Flood Estimation

Boughton et al. (1999) developed a Continuous Simulation System (CSS) for estimation of design floods, and applied this to a number of catchments of mid to small sizes in Victoria. The CSS comprised of a stochastic rainfall generator, the AWBM water balance model and a hydrograph model. The stochastic rainfall generator was based on Transition Probability Matrix model to generate daily rainfalls, and these were then disaggregated to hourly data. A multi objective calibration strategy was used to calibrate the runoff-routing model against the monthly runoff volume and maximum values of daily flow. To reduce the computational time, the model was run at daily time step during the long relatively dry periods and hourly time step during the storm event. They estimated the design flood values to 0.05% AEP and showed that the derived frequency curve calculated by the method was able to properly match the observed flood frequency curve for more frequent floods (5% AEP).

Newton and Walton (2000) further applied the CSS in a large (13 000 km^2), semi-arid catchment in Western Australia. They compared the design estimates produced by the CSS to the observed flood frequency curve and found that the design flood estimates overestimated the observed flood frequency curve for more frequent floods. They speculated that the discrepancy between observed flood frequency curve and the CSS result might be due to the sampling problem; the observed flood frequency curve was estimated based on a shorter period (31 years) of data, while the rainfall generation model was calibrated to longer (93 years) data. The observed streamflow data covered a relatively dry period and did not represent the total climatic variability over a longer period.

There have been other applications of Continuous Simulation approaches for estimation of the derived flood frequency curve, for example Haberlandt and Radtke (2013), Cameron et al. (1999) and Droop and Boughton (2002), to catchments of various sizes and characteristics. In all cases stochastic rainfall generators were used to extend the rainfall data. Although different rainfall generation and process models were used, all report that the derived distribution curve produced by the method was able to provide a satisfactory match to the observed flood frequency curves for large floods. However, in all cases described, the ability of the model to properly reproduce extreme flood events has not been confirmed, due to lack of data for extreme events.

3.4. Hybrid Continuous Event-Based Simulation

There is a range of “hybrid” approaches that do not fit neatly into the foregoing categories. Typically, hybrid approaches use statistical information on rainfall storms in combination with continuous simulation and event-based models. With this approach, long-term recorded (or stochastically generated) climate sequences might be used in combination with a continuous simulation model to produce a time series of catchment soil moisture and streamflows (which also may include simulation of snowpack conditions). This information is used to specify antecedent conditions for an event-based model, which is then used in combination with statistical information on rainfall storms to generate extreme flood hydrographs. For example, the SEFM model (MGS Engineering Consultants, 2009) undertakes soil moisture accounting and snowpack modelling for an extended period prior to the onset of an event to establish antecedent conditions, then uses a flood event model in combination with probabilistic design rainfall intensities to simulate the flood hydrographs.

SCHADEX (Paquet et al., 2013) is also an example of a hybrid approach. SCHADEX is a semi-continuous runoff-routing model in which a continuous hydrological simulation model is used to generate the possible hydrological states of the catchment, and floods are simulated on an event basis. The method incorporates a statistical model to characterise the distribution of rainfalls, where the observed rainfall series is split into several homogeneous sub-samples based on a classification of regional weather characteristics. The MORDOR hydrological model is used to convert rainfalls into floods; this is a conceptual, lumped, reservoir model with daily areal rainfall and air temperature as the driving input data. The principal hydrological processes represented are evapotranspiration, direct and indirect runoff, groundwater, snow accumulation and melt, and routing. Selected daily rainfalls are replaced by a synthetic generator for extreme rainfall estimation (Garavaglia et al., 2010), and the resulting daily discharge volumes are converted to peak flows using an empirical function derived from observed hydrographs. The results are fitted to a frequency distribution and used to derive flood quantiles typically out to 1 in 1000 AEP.

3.5. Performance of Methods

Ling et al. (2015) tested the Monte Carlo and Ensemble Event approaches using ten natural test catchments located in different areas of Australia, and the Continuous Simulation approach was applied to five of these catchments. [It should be noted that Ling et al. (2015) used the term “design event” to denote the use of an event model with a sample of temporal patterns, which corresponds to the Ensemble Event approach as described in Book 4, Chapter 3, Section 2; they did not test the deterministic “Simple Event” method as described in Book 4, Chapter 3, Section 2]. The catchments were selected to cover a range of climatic conditions, catchment sizes and catchment characteristics. Monte Carlo and Ensemble Event models were developed for each of the ten catchments and calibrated using observed rainfall and flow data. Three continuous simulation models were considered, the Australian Water Balance Model (AWBM, (Boughton and Droop, 2003)), SIMHYD (Chiew and McMahon, 2002) and GR4H (Mathevet, 2005).

The results of the event-based modelling showed that in general an initial loss-continuing loss model run using both the Monte Carlo and Ensemble approaches performed well in reproducing the at-site flood frequency curve over the range of catchments tested, over a range from 50% to 1% AEP. The exception to this was that the Monte Carlo model did not perform well for one catchment (located in the south-west of Western Australia) where the flow response to rainfall events varied widely. SWMOD (Water and Rivers Commission, 2003) was used as an alternative loss model for this catchment, and it was found that use of this model improved the results significantly over the initial loss-continuing loss model.

Sih et al. (2008) also evaluated the performance of Monte Carlo and Ensemble Event approaches, and they included comparison with the traditional Simple Event method. They tested the three methods on seven catchments covering the temperate and tropical regions of Australia, and considered both long duration (24 hours and longer) and short duration (less than 6 hour) storms. The Simple Event method was found to generally underestimate the peak flows for events. On the basis of the seven catchments considered, the Simple Event method underestimated the Monte Carlo solution by around 10% to 15%, although in some cases the method underestimated peak flows by between 50% to 70%. Sih et al. (2008) found much closer agreement between the Ensemble Event and Monte Carlo approaches, where generally the Ensemble Event method was found to underestimate the Monte Carlo solution by around 5%.

The results of the method testing on continuous simulation models by Ling et al. (2015) found that while it was possible to calibrate the models to reproduce the overall flow regime of the catchments, the highest flow peaks were markedly underestimated and the simulated flood frequency curve calculated from simulated Annual Maximum Series provided a very poor fit to the observed flood frequency curve. Weighting the calibration to the largest events in the series reduced the ability of the model to reproduce the overall flow regime, and provided only slight improvements in the accuracy of the derived frequency curves. It was found that the models could be calibrated directly to selected quantiles of the observed flood frequency curve, but this resulted in a very poor representation of hydrograph behaviour and large biases in flood volume. This testing clearly illustrated the multi-criteria nature of the calibration problem (Gupta et al, 1998), and showed that it is difficult to obtain a very good fit to both the flood frequency curve and hydrograph behaviour. Furthermore, comparison of the calibrated parameters resulting from the different calibration approaches also showed large differences in values, indicating a trade-off between reproducing the hydrograph and the best representation of the flood frequency curve.

Ling et al. (2015) investigated the effect of record length on model performance. The results from the two test catchments tested by Ling et al. (2015) found that even when twenty years of data is available at a site, the model results can vary significantly based on the period of record used in analysis. This is particularly evident when one period is noticeably drier or wetter than the other. This highlights the need to investigate how representative the available flow data is in the context of any available long-term rainfall records. Both the Monte Carlo and Ensemble Event approaches gave similar results.

Ling et al. (2015) also investigated the efficacy of applying the methods to ungauged catchments. The results of the investigation by Ling et al. (2015) illustrated that even when data is available from a neighboring gauged catchment, care must be taken in transposing inputs and parameters from similar gauged catchments. When parameters were transferred between models from dissimilar catchments, the results of both the Monte Carlo and Ensemble Event approaches were very poor. From these tests it is concluded that only catchments with similar climatic conditions, catchment sizes and catchment characteristics should be considered for providing model parameters for ungauged catchments.

3.6. Advantages and Limitations

An overview of the advantages and limitations of the different approaches to flood estimation is provided in Book 1, Chapter 3, though it is worth emphasizing some points here that are specific to the methods discussed in the Event Based Approaches and Hybrid Continuous Event-Based Simulation Sections above.

The Simple Event method has been the most commonly used approach to date in Australia. It is simple to apply, and information on the required design inputs - design rainfalls, single temporal patterns of average variability, and median design losses - are readily available for most locations in Australia. The probability neutrality assumption is maintained by selecting single “representative” values of the inputs; however, without independent information there is no way of knowing whether this assumption has been satisfied. Thus, while simple and easy to apply, the method is lacking in robustness and defensibility.

The Ensemble Event method represents a modest increase in complexity. Rather than undertaking a single run for each combination of event AEP and duration, it is necessary to undertake ten or so simulations and average the outcome; if single hydrographs are required for design purposes then these can be obtained by simple scaling of a hydrograph obtained from a representative event. The method does involve a little more tedium for practitioners, though most modelling software can be configured for batch processing, and the additional computation burden is of no consequence. The method is most readily suited to the consideration of temporal patterns, where testing has shown that in natural catchments it yields similar estimates to those derived from more rigorous approaches. While the approach represents an appreciable improvement over Simple Event methods, the approach does suffer from the limitation that it is not well suited to considering the influence of additional stochastic factors that may have an influence on the derived flood estimates. In natural catchments this includes the estimation of floods which are heavily influenced by the joint occurrence of highly variable losses and temporal patterns, catchments in which natural lake (or snowpack) levels are subject to variable antecedent conditions, or catchments where it necessary to consider seasonal variation in individual inputs. In disturbed catchments the method is unable to consider the influence of variable initial reservoir levels on dam outflows, the likelihood of debris blocking culverts and bridge waterway areas, or the influence of controlled discharges from infrastructure works that may be subject to some variability.

In contrast, Monte Carlo methods are well suited to the consideration of multiple sources of variability from natural or anthropogenic sources. Once the simulation scheme has been established, it is easily expanded to consider additional factors of importance. For example, the same sampling scheme can be used to accommodate the variability associated with seasonality of storm occurrence or temporal patterns, drawdown in a reservoir, or blockage factors. The information required to characterise aleatory uncertainty (ie. hydrologic variability) is often available in the historic record: if there is sufficient information available to simulate a process with a deterministic model, then the necessary information required to characterise variability can be readily obtained (or generated). Importantly, it is a simple matter to expand a simulation scheme to allow for correlations between the stochastic factors modelled. Thus, if there is information available that suggests that the dominant season is dependent on event severity, or that the available airspace in a reservoir decreases with event severity, then this is easily accommodated by using a conditional sampling scheme. The limitation of the method is that specialist modelling skills are required to develop bespoke Monte Carlo schemes, and that additional effort is required to ensure that the distributions used to characterise variability are appropriate for the conditions being simulated. The method can be expanded to include consideration of epistemic uncertainty (e.g. uncertainty in the routing parameters or in the design estimate of rainfall depth), but the necessary information for such schemes can be difficult to obtain and justify.

If the catchment is subject to complex interactions between stochastic factors and/or antecedent conditions, then consideration should be given to use of the Continuous Simulation approach. This method is particularly suited to the analysis of volume-dependent problems which are influenced by the interaction between multiple factors. For example, the analysis of peak levels at multiple points in a catchment that is influenced by hydraulic controls or which contain a cascade of storages. The use of Continuous Simulation approaches in these cases obviates the need to explicitly consider the manner in which factors combine, and if a long enough sequence is considered then it implicitly accounts for the joint probabilities involved. This approach also lends itself to the analysis of systems which are influenced by long duration events or sequences of flood events. Its limitation, however, is that the models most commonly used for Continuous Simulation are not well suited to representing the flood response in a catchment, particularly for rarer events. It is difficult to calibrate (then validate) a continuous model in a manner that adequately captures the sequencing and variability of streamflows while reproducing the behaviour that determines peak and volume of flood events. For estimating rare events, it is also necessary to calibrate and apply a suitable stochastic climate generator.

Hybrid models have the potential to combine the benefits of both continuous and event approaches, though at this stage insufficient investigations have been undertaken to determine whether such schemes provide demonstrable benefits over other approaches.

3.7. Example - Delatite River

The Delatite River is located in central Victoria and has a catchment area of 368 km^2. The catchment headwaters are located between Mount Buller and Mount Stirling in the Great Dividing Range. The river flows generally westwards through forests which become less dense as the river descends and then flows into Lake Eildon. The river descends a total of 1230 m over its 85 km length. A map of the catchment and its drainage network is shown in Figure 4.3.3 which also shows the schematic of a conceptual runoff-routing model developed for the catchment. Streamflow data is available at the Tonga Bridge gauging site (Gauge No. 405214) from March 1957 to date.

The runoff-routing model was fitted to three historic flood events, and the results for the largest event (September 2010) are also shown in Figure 4.3.3. The initial loss parameters fitted to the three events were 25, 10, and 15 mm, and the corresponding continuing loss parameters were 2.5, 1.5, and 2.5 mm/hr.

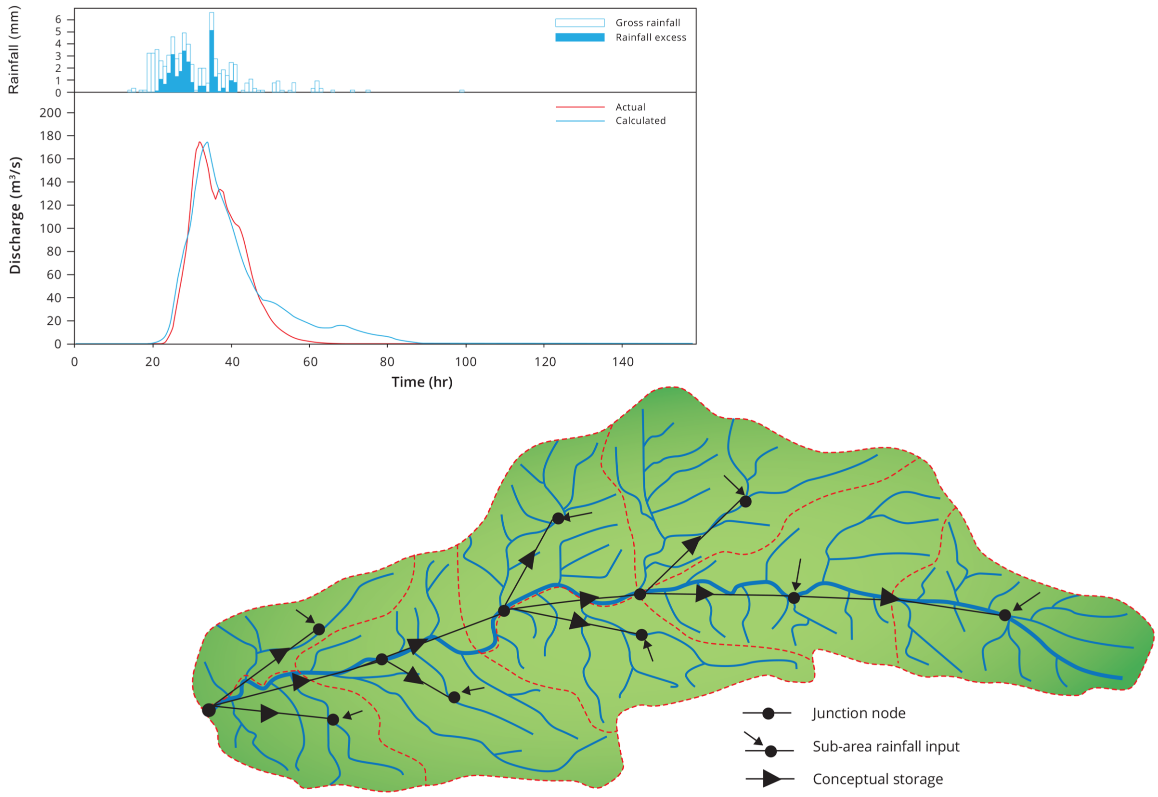

Figure 4.3.3. Schematic Layout of Delatite River catchment and Calibration to December 2010 Event

Three different approaches were used to derive design estimates using the calibrated runoff-routing model. The Simple Event approach used a single temporal pattern of average variability, along with a single set of loss parameters obtained from calibration to the three historic events. The Ensemble Event approach replaced the single temporal pattern with a sample of 19 patterns derived from rainfall events that have occurred in the inland region of south-east Australia, and used the same loss parameters as used in the Simple Event method. Monte Carlo results were obtained using the same set of temporal patterns as used in the Ensemble Event approach; the continuing loss parameter was held constant, and the initial loss was sampled from a non-dimensional distribution of initial losses (Hill et al., 2015) with a median loss value set equal to the value adopted for the Simple Event method. The results from these three approaches are shown in Figure 4.3.4 where it is seen that the Monte Carlo approach yields estimates that are very similar to the quantiles obtained from Flood Frequency Analysis. The Ensemble Event estimates are similar to but lower than those obtained using Monte Carlo analysis, and the Simple Event estimates are substantially higher. It is worth noting that all design flood estimates rarer than about 5% AEP lie within the confidence limits associated with the Flood Frequency Analysis.

Also shown in Figure 4.3.4 are the results obtained from Continuous Simulation. A number of conceptual models were trialled and the Sacramento model (Burnash et al., 1973) was found to provide the best results. Rainfall inputs to the model were obtained using gridded rainfall data (Jones et al., 2009) and mean monthly areal potential evapotranspiration inputs were obtained from the Bureau of Meteorology (Chiew et al., 2002). The model was initially calibrated to daily streamflows using 20 years of historic data, and then adjusted to reproduce the instantaneous peak flows over the same period. The model was used to derive 101 years of simulated streamflows using the gridded rainfall data, and a Generalised

Extreme Value distribution was then fitted to the annual maxima extracted from the time series. The results are shown in Figure 4.3.4, where it is seen that the design estimates are substantially lower than the results obtained from the event-based approaches. The derived flood frequency curve generally lies along the lower confidence limits of the frequency curve fitted using gauged maxima.

While no general conclusions should be drawn from this example about the relative efficacy of the different methods used, the results do illustrate the range of estimates obtained for a well gauged catchment. They indicate the degree of ‘model uncertainty’ that generally remains unknown when only a single simulation method is employed. The largest event used to fit the runoff-routing model occurred in December 2010 and has a peak similar in magnitude to the 2% AEP event determined from Flood Frequency Analysis. The period of record used to calibrate the Sacramento model spanned a representative range of climatic conditions. The data used in this example is more than is typically available, and nevertheless the design estimates vary by about a factor of two.

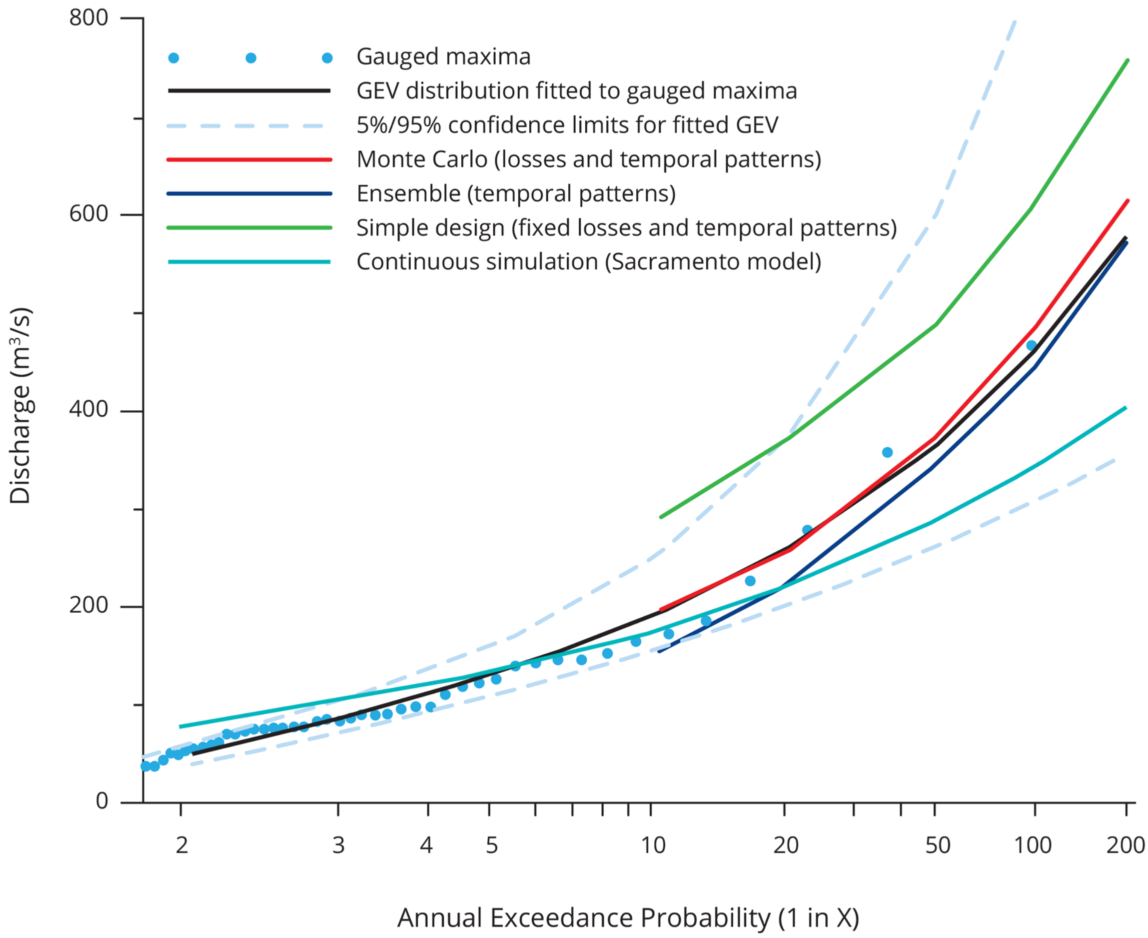

Figure 4.3.4. Comparison of Design Flood Estimates with Flood Frequency Curve for the Delatite River at Tonga Bridge

3.8. References

Abbott, M.B., Bathurst, J.C., Cunge, J.A., O'Connell P.E. and Rasmussen, J. (1986), An introduction to the European hydrological system- Systeme Hydrologique Europeen, SHE, 1: history and philosophy of a physically-based, distributed modelling system. Journal of Hydrology, volume 87(1-2), pp.45-59.

Beven K. J., Calver A. and Morris E. M. (1987). The Institute of Hydrology Distributed Model. Technical Report No. 98 Institute of Hydrology, Wallingford, UK.

Boughton, W.C. (1999), A daily rainfall generating model for water yield and flood studiesRep. Report 99/9, 21pp pp, CRC for Catchment Hydrology, Monash University, Melbourne.

Boughton, W. and Droop, O. (2003), Continuous simulation for design flood estimation-a review. Environmental Modelling & Software, 18(4), 309-318. Available at: http://linkinghub.elsevier.com/retrieve/pii/S1364815203000045 [Accessed October 3, 2013].

Boughton, W.C., Muncaster, S.H., Srikanthan, R., Weinmann, P.E., Mein, R.G. (1999) Continuous Simulation for Design Flood Estimation - a Workable Approach. Water 99: Joint Congress; 25th Hydrology & Water Resources Symposium, 2nd International Conference on Water Resources & Environment Research; Handbook and Proceedings; pp: 178-183.

Burnash, R.J.C., Ferral, R.L. and McGuire, R.A. (1973), A Generalised Streamflow Simulation System - Conceptual Modeling for Digital Computers. U.S. Department of Commerce, National Weather Service and State of California, Department of Water Resources.

Cameron, D.S, Beven, K.J, Tawn, J, Blazkova, S. and Naden, P. (1999), Flood frequency estimation by Continuous Simulation for a gauged upland catchment (with uncertainty). Journal of Hydrology, 219(3-4), 169-187. Available at: http://linkinghub.elsevier.com/retrieve/pii/S0022169499000578.

Chiew, F.H.S. and McMahon, T.A. (2002), Modelling the impacts of climate change on Australian streamflow. Hydrol. Processes, 16: 1235-1245.

Chiew, F.H.S., Wang, Q.J., McConachy, F., James, R., Wright, Q. and deHoedt, G. (2002), Evapotranspiration maps for Australia. Hydrology and Water Resources Symposium, Melbourne, May 20-23 2002.

Droop, O. and Boughton, W. (2002), Integration of WBNM into a continuous simulation system for design flood estimation. Research Report 2/02, Gilbert & Associates, Brisbane.

Feldman, A.D. (2000), Hydrologic Modeling System HEC-HMS. Technical Reference Manual. US Army Corps of Engineers (USACE), Hydrologic Engineering Center, HEC. Davis, CA, USA.

Garavaglia, F., Gailhard, J., Paquet, E., Lang, M., Garcon, R. and Bernardara, P. (2010), Introducing a rainfall compound distribution model based on weather patterns sub-sampling Hydrol. Earth Syst. Sci., 14: 951-964.

Green, J.H., Walland, D.J., Nandakumar, N. and Nathan, R.J. (2003), The Impact of Temporal Patterns on the Derivation of PMPDF and PMF Estimates in the GTSM Region of Australia. Presented at Hydrology and Water Resources Symposium, Wollongong, NSW, November 2003.

Gupta, V.K. and Sorooshian, S. (1985), The relationship between data and the precision of parameter estimates of hydrologic models. Journal of Hydrology, 81: 57-77.

Gupta, H.V., Sorooshian, S. and Yapo, P.O. (1998), Toward improved calibration of hydrological models: multiple and noncommensurable measures of information. Water Resour. Res., 34(4), 751–763.

Haberlandt, U. and Radtke, I. (2013), Derived flood frequency analysis using different model calibration strategies based on various types of rainfall-runoff data - a comparison. Hydrology and Earth System Sciences Discussions, 10(8), 10379-10417. Available at: http://www.hydrol-earth-syst-sci-discuss.net/10/10379/2013/ [Accessed October 13, 2013].

Heneker, T.M., Lambert, M.F. and Kuczera, G. (2001), A point rainfall model for risk-based design, J Hydrology, 247(1-2), 54-71.

Hill, P.I., Graszkiewicz, Z., Sih, K., Nathan, R.J., Loveridge, M. and Rahman, A. (2012), Outcomes from a pilot study on modelling losses for design flood estimation. Hydrology and Water Resources Symposium 2012, Sydney.

Hill, P.I., Graszkiewicz, Z., Loveridge, M., Nathan, R.J. and Scorah, M. (2015), Analysis of loss values for Australian rural catchments to underpin ARR guidance. Hydrology and Water Resources Symposium 2015. Hobart. 9-10 December 2015.

Hoang, T.M.T., Rahman, A., Weinmann, P.E., Laurenson, E.M. and Nathan, R.J. (1999), Joint Probability Description of Design Rainfalls. 25th Hydrology and Water Resources Symposium, Brisbane, 2009, Brisbane.

Jones, D. A., Wang, W. and Fawcett, R. (2009), High-quality spatial climate data-sets for Australia. Australian Meteorological and Oceanographic Journal, 5: 233-248.

Kuczera, G., Lambert, M., Heneker, T., Jennings, S., Frost, A. and Coombes, P. (2006), Joint probability and design storms at the crossroads. In 28th International Hydrology and Water Resources Symposium, Woolongong, pp: 10-14.

Ling, F., Pokhrel, P., Cohen, W., Peterson, J., Blundy, S. and Robinson, K. (2015), Australian Rainfall and Runoff Project 12 - Selection of Approach and Project 8 - Use of Continuous Simulation for Design Flow Determination, Stage 3 Report.

Linsley, R.K., Crawford, N.H. (1960), Computation of a synthetic streamflow record on a digital computer. In Int. Assoc. Sci. Hydrol., pp: 526-538.

MGS Engineering Consultants (2009), General Stochastic Event Flood Model (SEFM), Technical Support Model. Manual prepared for the United States Department of Interior, Bureau of Reclamation Flood Hydrology Group.

Mathevet, T. (2005), Which rainfall-runoff model at the hourly time-step? Empirical development and intercomparison of rainfall-runoff models on a large sample of watersheds. ENGREF, Paris, FRANCE.

Nathan, R.J. (1992), The derivation of design temporal patterns for use with generalized estimates of Probable Maximum Precipitation. Civil Engng. Trans., I.E. Aust., CE34(2), 139-150.

Nathan, R.J. and Weinmann, P.E. (2004), An improved framework for the characterisation of extreme floods and for the assessment of dam safety. Hydrology: Science & Practice for the 21st Century, Vol 1. Proc. British Hydrol. Soc., London, pp: 186-193.

Nathan, R.J., Weinmann, P.E. and Hill, P.I. (2002), Use Of A Monte Carlo Framework To Characterise Hydrological Risk, ANCOLD Bulletin, (122), 55-64.

Nathan, R., Weinmann, E., Hill, P. (2003), Use of Monte Carlo Simulation to Estimate the Expected Probability of Large to Extreme Floods, proc. 28th Int. Hydrology and Water Res. Symp., Wollongong, pp: 1.105-1.112.

Newton, D. and Walton, R. (2000), Continuous Simulation for Design Flood Estimation in the Moore River Catchment, Western Australia. In Hydro 2000 Hydrology and Water Resources Symposium. Canberra, ACT, pp: 475-480.

Paquet, E., Garavaglia, F., Gailhard, J. and Garçon, R. (2013), The SCHADEX method: a semi-continuous rainfall-runoff simulation for extreme flood estimation, J Hydrol., 495: 23-37.

Pilgrim, DH (ed) (1987) Australian Rainfall and Runoff - A Guide to Flood Estimation, Institution of Engineers, Australia, Barton, ACT, 1987.

Pilgrim, D.H. and Cordery, I. (1975), Rainfall temporal patterns for flood designs, Proc. Amer. Soc. Civ. Engrs., J Hydrauclis Div., 100(1), 81-95.

Pilgrim, D.H., Cordery, I. and French, R. (1969), Temporal patterns of design rainfall for Sydney, Institution of Engineers, Australia, Civil Eng. Trans., CE11: 9-14.

Rahman, A., Hoang, T M.T., Weinmann, P.E. and Laurenson, E.M. (1998), Joint probability approaches to design flood estimation: a review. CRC for Catchment Hydrology, Report 199808, Melbourne, Australia.

Rahman, A., Weinmann, P.E. and Mein, R.G. (2002a), The use of probability-distributed initial losses in design flood estimation. Australian Journal of Water Resources. 6(1), 17-29.

Rahman, A., Weinmann, P., Hoang, T.M. and Laurenson, E. (2002b), Monte Carlo simulation of flood frequency curves from rainfall. Journal of Hydrology, 256(3-4), 196-210. http://doi.org/10.1016/S0022-1694(01)00533-9.

Schaefer, M.G. and Barker, B.L. (2002), Stochastic Event Flood Model (SEFM), Chapter 20, Mathematical Models of Small Watershed Hydrology and Applications, Water Resources Publications, LLC, VJ Singh editor.

Schaefer, M.G. and Barker, B.L. (2004), Stochastic Event Flood Model (SEFM) - Users Manual, MGS Engineering Consultants Inc, prepared for US Bureau of Reclamation Flood Hydrology Group.

Sih, K., Hill, P.I., Nathan, R.J., (2008), Evaluation of simple approaches to incorporating variability in design temporal patterns. Water Down Under 2008 (incorporating 31st Engineers Australia Hydrology and Water Resources Symposium), pp: 1049-1059.

Singh, S.K. and Bárdossy, A. (2012), Calibration of hydrological models on hydrologically unusual events. Advances in Water Resources, 38: 81-91.

Srikanthan, R. and McMahon, T.A. (2001), Stochastic generation of annual, monthly and daily climate data: A review. Hydrology and Earth System Sciences, 5(4), 653-670, Available at: http://www.hydrol-earth-syst-sci.net/5/653/2001/.

Vaze, J., Jordan, P., Beecham, R., Frost, A., Summerell, G. (2012), Guidelines for rainfall-runoff modelling Towards best practice model application.

Water and Rivers Commission (2003), SWMOD Version 2 A Rainfall Loss Model for Calculating Rainfall Excess, User Manual (Version 2.11).

Weinmann, P.E., Rahman, A., Hoang, T.M., Laurenson, E.M. and Nathan, R.J. (2000), Monte Carlo Simulation of flood frequency curves. Proc., Hydro 2000, 3rd International Hydrology and Water Resources Symposium, IE Aust, pp: 564-569.

Weinmann, P.E., Rahman, A., Hoang, T.M.T., Laurenson, E.M. and Nathan, R.J. (2002), Monte Carlo Simulation of Flood Frequency Curves from Rainfall - The Way Ahead. Aust. Journal of Water Resources, 6(1), 71-79.

Chapter 4. Treatment of Joint Probability

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4.1. Introduction

In many applications of flood simulation it is necessary to understand and apply the basic probability concepts involved when a range of factors combine to produce a flood event or when different events occur jointly. Such applications range from the stochastic simulation of design flood events allowing for the joint probabilities of several key flood producing or flood modifying factors, to typical situations where flood risk results from various combinations of flood events that have different causes or occur at different locations.

Book 4, Chapter 4, Section 2 introduces basic probability concepts that are applied in flood simulation methods and in determining flood risks for situations where several factors or events interact. It then describes typical practical applications where the interaction of different factors or events need to be considered and points to other sections where individual applications are treated in more detail. Book 4, Chapter 4, Section 3 is devoted to introducing Monte Carlo simulation as the most practical and flexible method of deriving distributions that result from the interaction of several stochastic components. Book 4, Chapter 4, Section 4 illustrates the application of joint probability concepts to a typical flood estimation problem

4.2. Probability Concepts

4.2.1. Variability and Uncertainty

When considering the variabilities of different factors involved producing flood risk and in the assessment of joint probabilities, it is worth differentiating between the *temporal and spatial variability* of the climate and hydrologic factors being modelled (aleatory uncertainty), and the random variation resulting from unavoidable *uncertainty* in the model inputs, structure, and parameters (epistemic uncertainty). Similar solution methods can be used to consider both these sources of uncertainty and thus there is sometimes some confusion about what aspects are being considered. However, the nature of the information available for these two broad sources of uncertainty – and hence the defensibility of the analyses undertaken – is markedly different.

Aleatory uncertainty represents the *natural variability* inherent in most hydrologic systems. In the context of design flood estimation, this usually involves consideration of natural variability in the characteristics of storm rainfalls (depths, temporal and spatial patterns), antecedent conditions (as they relate to initial losses, water levels in natural lake systems and snowpack characteristics), coincident streamflows (or levels) at the confluence of two streams, and the influence of tide levels on estuarine flood behaviour. Aleatory uncertainty associated with anthropogenic causes is also commonly a factor that needs to be considered. Perhaps the most common factor to be considered in design flood estimation is initial reservoir levels in dams (either singly or in cascade), though this can include consideration of the reliability of operating equipment (e.g. spillway gates and other forms of outlet works), and debris blockage of waterway areas provided for spillways, drainage works and bridges. Factors which vary randomly over time are termed stochastic variables.

Epistemic uncertainty, on the other hand, relates to the uncertainty arising from *a lack of knowledge* about hydrologic factors and their governing processes. In the context of design flood estimation, epistemic uncertainty is commonly associated with errors involved in rating curves (ie. in the relationship used to estimate streamflows from gauged levels), in the estimation of catchment rainfalls from point observations, and the uncertainties involved in estimating model parameters from a limited number of relevant events. An important source of epistemic uncertainty arises from the need for extrapolation. That is, there may be an adequate amount of information available at a particular site for estimating the exceedance probability of frequent floods, but additional uncertainty is introduced when transposing such information to an ungauged location, or when extrapolating to events much larger than have occurred in the historic record. As the degree of extrapolation increases, so does the uncertainty in the appropriateness of the configuration, or indeed of the conceptual structure, of the model being used. Such uncertainties arise from lack of knowledge, and as such can be reduced over time with collection of relevant data and increases in our understanding.

This Chapter only considers the influence of aleatory uncertainty on joint probability, and consideration of epistemic uncertainty is discussed in Book 1, Chapter 2 and Book 7, Chapter 9. The focus of this chapter is on the use of techniques that minimise the introduction of bias in the exceedance probability of the final design estimate. Such estimates will always contain uncertainty due to lack of knowledge, but the methods presented here are intended to make best use of the information on natural variability that we do have.

4.2.2. Joint and Conditional Probabilities

The range of situations or applications when combinations of different factors or events need to be considered can be grouped on the basis of the different probability concepts being applied.

4.2.2.1. Joint Occurrence of Different Factors or Events

In flood hydrology there are many situations where a number of factors need to be considered jointly when determining the probability of a flood outcome, in other words when “Event A” *AND* “Event B” determine the flood outcome. This includes the joint influence of a number of factors in determining the magnitude of a design flood event, e.g. the average depth and spatial/temporal distribution of rainfall inputs, the magnitude and temporal distribution of losses and the influence of flood modifying factors, such as the initial conditions of natural and artificial storages in the catchment. The flood simulation process then needs to allow for the joint probability of the different factors, which may be correlated or independent of each other.

The interaction of these different factors can be described by a *joint probability distribution* (Benjamin and Cornell, 1970; Haan, 1974). A *bivariate probability distribution* describes the joint probability of two variates *x* and *y*, and this case is the simplest to visualise (refer to Figure 4.4.1). Each of the two variables has a marginal probability distribution, $p(x)$ and $p(y)$, which represents the probability distribution without considering the influence of the other variable. At a particular value of one variable, say at x_0, the distribution of the other variable *y* can be said to be conditioned on *x* and this is referred to as the *conditional probability distribution* of *y*:

$$p(y|x = x_0) \tag{4.4.1}$$

The marginal distributions are illustrated in Figure 4.4.1 for the probability densities of a bivariate normal distribution in *x* and *y* (with means of 70 and 50, respectively), where the conditional probability distribution is shown for *x* = 90.

It is clear from the figure that the marginal probability distribution of *y* can be obtained by integrating the conditional probability distributions of *y* for all values of *x*. For independent events, the distribution of one variable is not conditioned on the other, and all conditional distributions are thus identical to the marginal distributions of that variable.

The concepts of marginal and conditional probability distributions can be extended to *multivariate probability distributions* where several variables are involved.

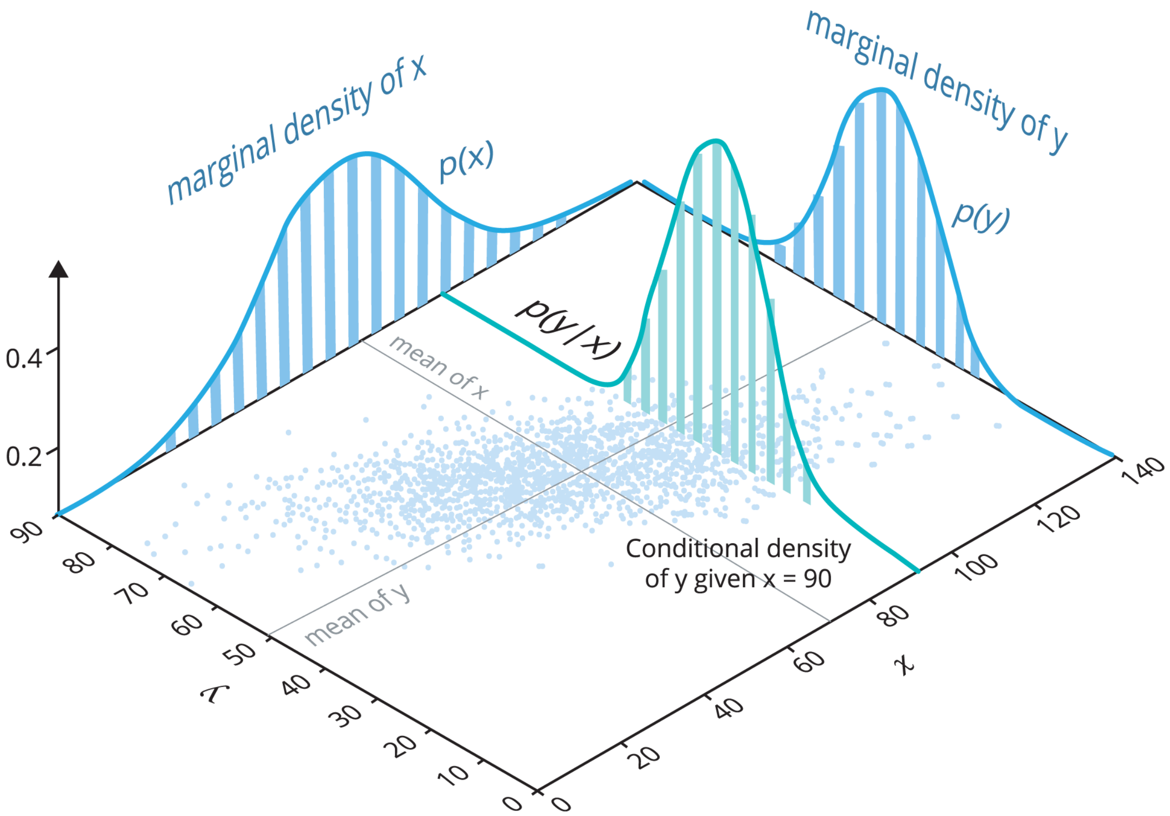

Figure 4.4.1. Joint Probability Density for a Bivariate Normal Distribution

The joint probability distribution concepts can also be applied to deal with the joint occurrence of events that are simulated separately. Examples of such applications include the interactions of riverine (or overland) flooding and sea level anomalies (Book 6, Chapter 5), the joint probability of reservoir inflows and initial storage contents, and the joint consideration of mainstream and tributary floods.

The general solution approach to joint probability problems and the selection of factors or events to be included in the joint probability framework are discussed in Sections Book 4, Chapter 4, Section 2 and Book 4, Chapter 4, Section 2 respectively.

Analytical approaches are available to deal with relatively simple joint probability applications. A special case is where component probability distributions can be considered to be independent of each other. In this case the joint probability can be evaluated simply by multiplying the component probabilities from the marginal distributions. However, in practice most joint probability applications are more complex and are most readily addressed by Monte Carlo simulation. In this approach the joint probability distribution is derived by randomly sampling from the (marginal) component distributions and simulating the system response a sufficient number of times to define the output distribution over the range of interest. The method can readily deal with several component distributions and correlations between them. This is the practical joint probability approach dealt with separately in Book 4, Chapter 4, Section 3 *Monte Carlo Simulation*.

Typical examples of practical problems are discussed in Book 4, Chapter 4, Section 2 *Typical Joint Probability Applications*, and this includes references to solutions that do not require practitioners to develop their own solution framework.

4.2.2.2. Combination of Conditional Occurrences

There are flood estimation applications where it is most practical or efficient to partition the total range of a key variable into a number of segments or intervals.

A typical example is to divide the range of rainfall input magnitudes into a number of intervals and then calculating the probability of a particular flood outcome conditional on this range of rainfall inputs. Key variables for other flood estimation applications may also be partitioned in a similar way.

The marginal exceedance probability of the flood outcome of interest *X* can then be calculated by the application of the Total Probability Theorem (Haan, 1974):

$$P(X > x) = \sum_i P\left[X > x \middle| C_i\right] p\left[C_i\right] \tag{4.4.2}$$

where the term $P\left[X > x \middle| C_i\right]$ denotes the conditional probability that the flood outcome *X* generated from this interval C_i exceeds *x* and the term $p\left[C_i\right]$ represents the probability that the conditioning variable falls within the interval *i*. For Equation (4.4.2) to be applicable, the set of conditioning events C_i needs to be mutually exclusive (meaning no overlap) and collectively exhaustive (meaning that the probabilities of the conditioning events have to add up to 1.0).

Typical applications of conditional probability concepts and the Total Probability Theorem are further discussed in Book 4, Chapter 4, Section 2.

4.2.2.3. Combination of Separate Independent Events

A specific flood outcome, such as flooding above the floor level of a building or flooding above a certain threshold level where access to a property is cut, may occur as a consequence of different events whose occurrences may be considered to be independent of each other. An example of such separate events is flooding as a result of high river levels (Event A) and flooding caused by overflows from a local drainage system (Event B). If the river flooding typically occurs from an extensive storm system over a large catchment and the drainage flooding from thunderstorms over a small local catchment, then these events can be considered to be essentially independent.

The combined exceedance probability of this specific flood outcome from either Event A *OR* Event B can then be calculated as:

$$P(A+B) = P(A) + P(B) - P(A)P(B) \tag{4.4.3}$$

where P(A)P(B) represents the exceedance probability of Events A and B occurring together. For events of relatively small AEPs this product is quite small and can generally be neglected. The combined exceedance probability of several events can be evaluated in an analogous fashion. An example involving several events is when flood frequency curves for different seasons are combined to determine the annual frequency curve.

It is important to note that for Equation (4.4.3) to be applicable, the different events being considered have to be defined in terms of the same magnitude (not exceedance probability).

In the example situations discussed above the interest is on the combined probability of occurrence of separate events. When the reliability of a linear structure such as a road or railway is being considered, the interest is not on the combined probability of exceedance of a given flood standard at different locations but on the combined *non-occurrence probability* or *survival probability*. Under the assumption of independent occurrences of damaging events at different locations, the overall reliability of the linear structure can be calculated as the product of the non-exceedance probabilities of a damaging event at different locations. The combined risk of failure of the structure can then be determined as the complement of the overall reliability.

Book 4, Chapter 4, Section 2 provides further discussion of this particular form of probability calculations.

4.2.3. Typical Joint Probability Applications

Floods by their nature are the result of the joint occurrence of different flood producing influences, and thus most practical problems require consideration of the joint probabilities involved. This section describes some typical examples of such problems, and provides references to some general and specific procedures for their solution.

It is commonly required to estimate flood risk downstream of a storage, where the outflow peak is dependent on the initial water level. If the variation in initial water level is small, such as in a retarding basin or small on-line storage, then it may appropriate to adopt a typical starting storage from the central range of conditions. However, the relationship between inflow and outflow can be highly non-linear, thus in general it cannot be expected that adoption of a mean initial water level will provide an unbiased estimate of outflows. A maximum water level could be used and justified on the basis that it provides a conservatively high estimate of flood risk, but introducing conservatism in intermediate steps of the analysis should generally be avoided as the compounding effects of such assumptions can undermine the validity of any risk-based decisions. If the initial water level does have an appreciable impact on the outflow flood, ie. when the available flood storage is large compared to the flood volume, then it will be necessary to give explicit consideration to the joint probabilities involved. Detailed guidance on this type of problem is provided in Book 8, Chapter 7, and worked examples using both analytical and numerical schemes are provided in Book 8, Chapter 8, Section 4. The general computational elements involved in the Monte Carlo solution to this type of problem are discussed in Book 4, Chapter 4, Section 3; particular attention is drawn to the need for conditional sampling (Book 4, Chapter 4, Section 3) as it possible that the storage level associated with a given exceedance probability tends toward a maximum value as the event magnitude increases.

Flood levels in estuarine regions may be dependent on the combined influence of storm surge and tide levels. The degree of influence depends on a number of factors, but the lower limits of such flood estimates are determined by assuming that fluvial flood levels are wholly independent of the ocean level; conversely, the upper limits of such flood estimates are derived using the assumption of complete dependence, that is, that fluvial floods will always coincide with ocean levels of the same exceedance probability. Book 6, Chapter 5 provides a practical approach to the solution of this class of problem. This guidance assists the practitioner determine whether consideration needs to be given to the dependence of flood levels on ocean conditions, and if so, then site-specific estimates for any location on the Australian coastline can be determined using a software tool (http://p18.arr-software.org/) based on the bivariate extreme value distribution. A Monte Carlo solution could be developed by generating correlated variates in combination with a stratified sampling scheme using the procedures described in Book 4, Chapter 4, Section 3 and the dependence parameters described in Book 6, Chapter 5. In concept, the spreadsheet based worked example presented in Book 4, Chapter 4, Section 4 is directly applicable to this type of problem, the only difference being that the correlation term relating to tributary flows replaces the dependence term governing coincident ocean levels. Regardless of the approach used, any solution of this type of problem will require the undertaking of deterministic modelling to obtain flood levels for different combinations of riverine flood and storm tides.

Another common problem arises when considering the influence of tributary flows at a confluence relevant to the region of interest. There are a number of solutions to this class of problem, and the degree of complexity required will dependent greatly on the sensitivity of the outcome to selected simplifying assumptions. If the focus is on mainstream flows, then it may be sufficient to estimate the tributary contribution by estimating the average flood inflow coincident with mainstream flow conditions; Book 8, Chapter 8, Section 5 presents a simple worked example for this based on the use of a bivariate log-Normal distribution. Conversely, if the focus is on tributary flows, then the assumption that there is an average flood in the mainstream that is coincident with local flooding is likely to yield a biased outcome. This is because any variation in mainstream floods may have a large influence on local flood levels, at least for the region susceptible to backwater influences. The worked example presented in Book 4, Chapter 4, Section 4 is directly applicable to this type of problem, the only difference in application is that levels computed using hydraulic modelling (final column of Table 4.4.2) relate to upstream levels in the tributary, rather than downstream of the confluence. It should be noted that the inputs to this worked example may be derived by either Flood Frequency Analysis or rainfall-based modelling. It would be expected that the deterministic relationship between mainstream flows and flood level is most easily obtained from some form of hydraulic modelling, but if gauged information is available for a range of historic events, then a suitable deterministic function may be obtained directly through analysis of the data, thus obviating the need for hydraulic modelling. An example of such an analysis is provided by Laurenson (1974).

The general form of solutions to the above problems all conform to the conceptual framework described in Book 4, Chapter 4, Section 3. The sub-sections following this framework provide for parametric and non-parametric approaches to characterising the input distributions, and allow for the additional consideration (if required) for dealing with conditional dependencies. The generic procedures covered here are intended to cover situations not specifically catered for in the methods presented elsewhere in ARR, as discussed above.

4.2.4. Typical Conditional Probability Applications

It is sometimes appropriate to estimate the probability of occurrence of a flood event subject to a restrictive range of conditions, such as the time of year or a specific range of rainfalls. If so, then additional steps are required to estimate the probability of exceedance for the complete range of conditions that might apply. It is common in hydrology to consider both conditional and unconditional probabilities, and care is required when interpreting and reporting such analyses to avoid confusion.

For example, conditional probability estimates are often required for the estimation of flood risk for construction activities. Flood risk varies seasonally throughout the year, and construction works may be scheduled to occur in a season of low flood risk. In this case it is appropriate to estimate conditional flood probabilities relevant to the particular season of interest; such analyses might involve undertaking Flood Frequency Analysis using flood maxima that have occurred over the months scheduled for construction, or else a rainfall-based approach might be used in which seasonal design rainfalls are used in combination with season-specific losses. The flood risk estimates derived from such analyses are *conditional* upon the season considered, and without additional analyses it is not possible to convert these estimates to annual risks.

The nature of the additional analyses required to derive unconditional estimates of annual risk depends on whether the conditioning events are *mutually exclusive* or not. Estimating annual flood risks based on seasonal analyses represents a mutually exclusive set of estimates, as clearly the annual maximum event cannot occur in two different seasons in the one year. Being mutually exclusive, the annual risk that a flood exceeds a given value is obtained by the simple addition of the individual seasonal exceedance probabilities.

It is often the case, however, that the conditioning events are not mutually exclusive. A common example of this is the estimation of flood immunity along a length of linear infrastructure, such as a major road or railway line. Here, the annual maximum event may well occur at multiple locations along its length, and thus the annual risk that access between two locations might be disrupted cannot be obtained by simply summing the estimates made at each individual crossing. Instead, some account must be given to the dependence of the factors that give rise to the individual floods. The probability of closure for an existing length of infrastructure is not simply equal to the exceedance probability of the most vulnerable crossing as this ignores the contribution of flood exceedance probabilities from rainfalls that may occur from other independent weather systems. Whether or not the degree of dependence needs to be considered depends on the significance of the outcome when the initiating events are considered to be wholly dependent or independent. The greater the difference between these two extremes, the greater is the need to complicate the solution by the explicit consideration of the dependencies involved.

Practitioners need to decide the appropriate level of complexity required to come up with a practical solution in a manner that is proportionate to the nature of the problem and the available resources. The simplest approach is to assume that the factors of most importance are highly correlated and that alternative combinations of conditions contribute little to the overall flood risk. With reference to Figure 4.4.2, it is seen that in temperate climates it might be expected that large long duration rainfall events occur at times when soil moisture is high and consequently catchment losses are low; conversely short duration (thunderstorm) events might occur when losses are high. As long as due care is given to matching the design inputs to match the dominant mechanism of interest, then it may be appropriate to derive estimates of rainfall-based flood estimates on an annual basis. Conversely, if the design loading of interest is sensitive to a mix of storm durations and catchment conditions, then it may be warranted to derive rainfall-based estimates on a seasonal basis and compute annual risks by summation of the seasonal exceedance probabilities.

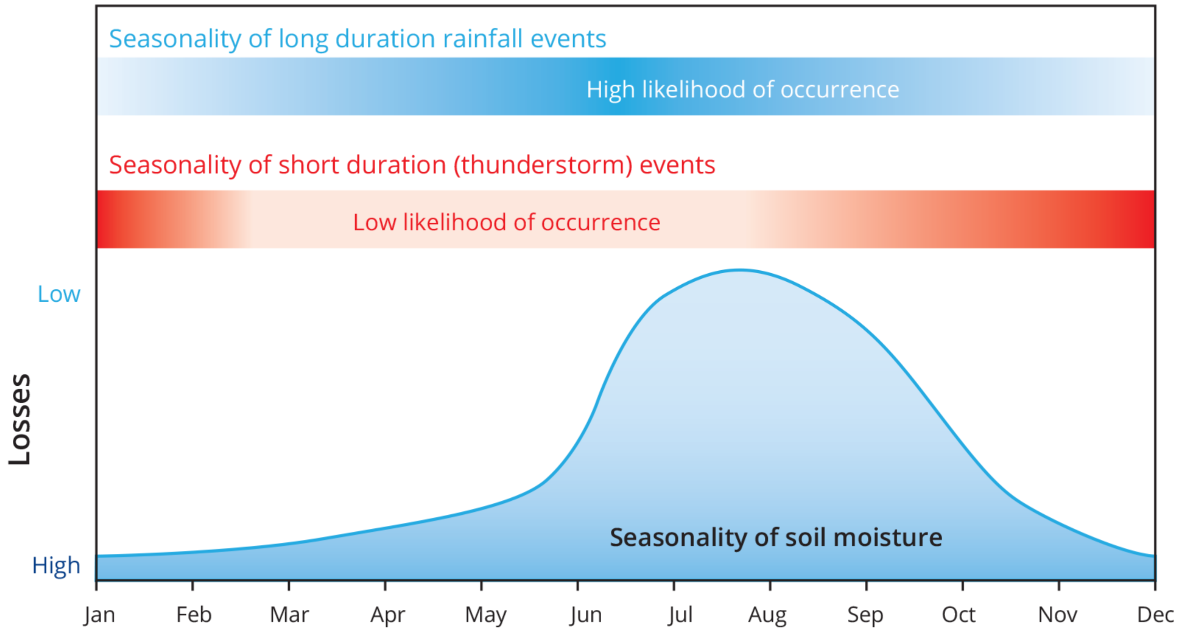

Figure 4.4.2. Difference in the Seasonal Likelihood of Large Long Duration Rainfall Events and Large Short Dduration Rainfall Events and their Concurrence with Catchment Losses

The analytical approach required to accommodate conditional probabilities when the events are not mutually exclusive is more complex. There are a number of different approaches that can be used, and in any given design situation the best approach to adopt depends on the nature and importance of the problem. Monte Carlo simulation in combination with evaluation of the Total Probability Theorem provides a general solution to problems involving conditional probabilities, and details on how to undertake such an approach is provided in Book 7, Chapter 9. However, different approaches are often available and the choice of solution does somewhat depend on the skills and experience of the practitioner. For example, while the assessment of flood immunity along a length of linear infrastructure could be solved by generating correlated rainfall inputs for use with event-based models, the use of gridded rainfall fields in combination with continuous simulation obviates the need to explicitly consider the joint probabilities involved (Jordan et al., 2015). Other approximate approaches that explicitly consider correlation in rainfall events have also been applied (Fricke et al., 1983), and a simple analytical example demonstrating a similar approach is provided in Book 8, Chapter 7, Section 3 and Book 8, Chapter 8, Section 5.

The techniques presented in this Book can also be applied to events which are mutually exclusive, however again it may be appropriate to adopted simpler approaches. For example, a discussion of the specific issues involved in computing annual risks from analyses undertaken on a seasonal basis is provided in Book 8, Chapter 7, Section 4; this approach is applicable to any design in which the conditional contributions are mutually exclusive, where the relative importance of the different factors may vary with event severity.

4.2.5. General Approach

Catchment Modelling Systems used to derive flood estimates can be considered to have stochastic and deterministic components. As discussed above, the stochastic components are related to factors (like rainfalls and losses) whose state at any given point in time is uncertain. The deterministic component represents processes that can be described mathematically and defines the manner in which inputs combine to yield a given output. This transformation is deterministic in the sense that the model will always yield the same outcome for a given set of inputs, antecedent conditions, and parameter values.

The general form of this concept is shown in Figure 4.4.3 for three different examples. In one example, the stochastic component represents the flood frequency distributions of two tributaries, where the deterministic component represents the manner in which the flows combine at their confluence. For a reservoir, the stochastic inputs might represent the frequency distribution of inflows and initial storage levels, where the deterministic component represents the relationship between inflows, storage and outflows. In hydraulic modelling, stochastic inputs may be used to represent inflows to a stream reach as well as the tide levels for a downstream boundary condition, where the deterministic component is governed by the hydraulic equations that predict flood level as a function of streamflow, reach characteristics and boundary conditions.

A variety of approaches are available for solving this general type of problem. Laurenson (1974) provides a general solution based on the matrix multiplication of a probability distribution of a stochastic input with a transition matrix derived from the deterministic operation of the system. The method is very general and suited to numerical solution. Careful effort is required to develop the elements of the transition matrix, and additional conditional probability terms need to be evaluated to allow for correlations in the inputs.

The joint occurrence of correlated stochastic factors can be evaluated using bivariate distributions, and there are numerous applications in the water resources literature where these have been used. The methodology used to assess the coincidence of catchment flooding and extreme storm surge for the coastline of Australia was developed using such an approach (Zheng et al., 2014), and is covered in detail in Book 6, Chapter 5. There are fewer examples where multivariate extreme distributions are used, and possibly the use of copula functions in combination with univariate distributions afford a more practical approach (Favre et al., 2004; Genest and Favre, 2007; Chen et al., 2012). Kilgore et al. (2010) reviews a range of methods and develop a general methodology for estimating joint probabilities of coincident flows at stream confluences based on the use of copulas which is intended for use by practitioners.

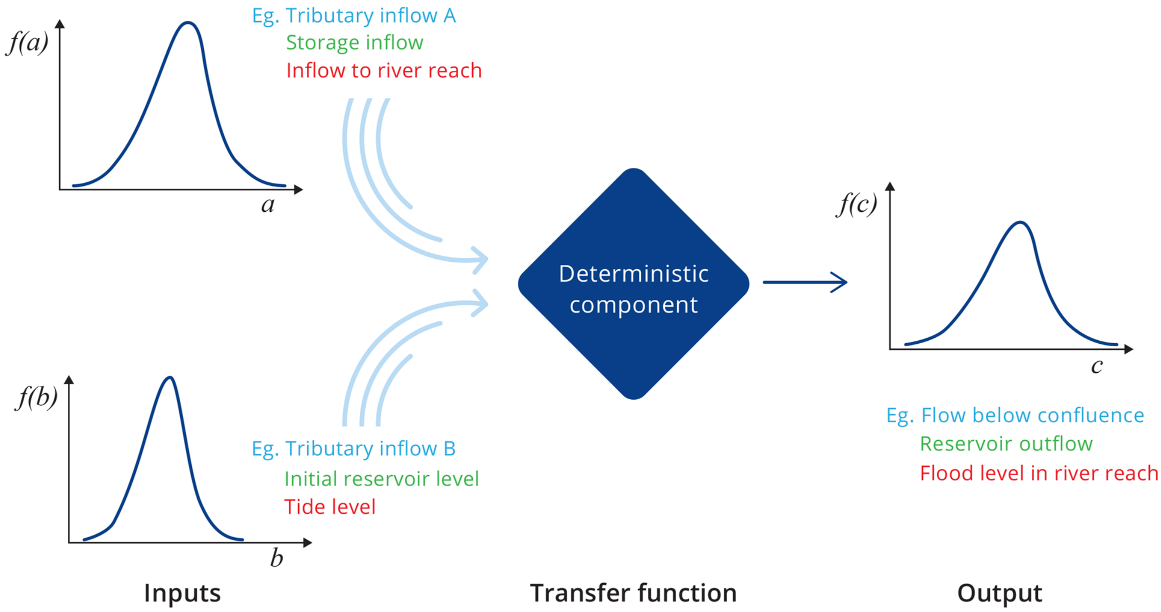

Figure 4.4.3. Generic Components that need to be Considered in Solution of Joint Probability Problems

However, the development and application of such approaches does require considerable statistical skill and they are not well suited for application by the majority of practitioners. Also, regardless of the methods used to characterise the extreme (possibly correlated) behaviour of the inputs, it is still necessary to model the deterministic component to determine how the various inputs combine to yield outputs of different magnitudes. Developing such response functions over the range of inputs required is itself a demanding task, and there is advantage if this can be done in such a way that leads directly to the exceedance probabilities of interest.

Monte Carlo techniques provide a structured means to generating outputs for a wide range of inputs, and if formulated correctly they represent a generic solution to the problem illustrated in Figure 4.4.3. With this approach, inputs are randomly sampled many hundreds, or thousands of times, and used in conjunction with a model of the deterministic component to obtain a distribution of the required outputs. Statistical analysis is then used to estimate the exceedance probability of the output variable of interest.

One of the main attractions of Monte Carlo methods is that the modelling tools and hydrologic concepts involved are essentially identical to those used in traditional approaches. Differences only arise in the manner in which the inputs are handled and the results analysed. Once the necessary framework has been developed, the factors of most importance can be modelled as stochastic inputs, and those of lesser importance can be set at fixed values. Many practitioners are used to developing automated means for running simulation models; such approaches can be adapted to Monte Carlo simulation by using simple probability models to generate the inputs, and straightforward statistics to analyse the outputs. The approach thus represents a powerful means of capturing the influence of variability on hydrologic systems in a manner that requires only a modest increase in the level of modelling sophistication.

4.2.6. Selection and Treatment of Factors

Any explicit analysis of joint probability should only focus on those factors which are characterised by a high degree of variability and which have a significant influence on flood response. Factors which have a small range of variation or a small influence on outcomes are best treated as fixed inputs to the model. The degree of importance of any factor can be assessed by simply undertaking a sensitivity analysis whereby the values of individual factors are varied systematically over the range of their expected variation and the factors with the largest stochastic influence are explicitly included in joint probability analysis. Some factors may have a large influence on the outcome (e.g. routing parameters) but are principally sources of epistemic uncertainty and thus do not need to be treated in a stochastic manner.

The common attribute of stochastic factors that influence flood response is that at any given point in time their state is uncertain. With sufficient data it is possible to estimate their average state and other characteristics related to their range and variability, and possibly the nature of their dependence on the magnitude of other factors. Often natural factors vary in a systematic fashion with the time of day or season, and they may be correlated. For example, initial loss might range between 0.1 and five times its median value but 70% of the time it might range between 0.5 and 1.5 times the median; average summer losses might also be expected to be twice the magnitude of winter losses, and because of the likelihood of rainfall occurring before intense rainfalls bursts, it might be that initial loss values vary inversely (ie. are negatively correlated) with rainfall depth.

4.2.6.1. Use of Regionalisation and Standardisation

Information on the variability and dependence of hydrological factors can be obtained from regional or catchment-specific (“at-site”) data. Physical reasoning should be used to determine what sources of data might be relevant to the catchment of interest. For example, information on the temporal variability of storm rainfalls is associated with storm types which may occur over a large region, and thus rainfall data collected over an extensive geographic area can be used to obtain information on the variability of temporal patterns that are relevant to a specific catchment (Book 2, Chapter 5). Conversely, the spatial variability of rainfalls across a catchment is subject to natural variability arising from storm behaviour, but it might be expected that there is a systematic component to this that is dependent on local topography and the dominant storm direction; accordingly, local rainfall data should be used to characterise catchment-specific spatial variability.

When considering the use of regional information it is often useful to standardise the data in some form to allow transposition from one site to another. An example of this relevant to flood estimation is the distribution of losses, as illustrated in Figure 4.4.4. While the typical magnitude of losses varies from one catchment to another, standardising these values (by simply dividing by the median value for the catchment) reveals that the likelihood that the catchment is wetter or drier relative to typical conditions is similar for a wide variety of catchment types (Hill et al., 2015). The representation of temporal pattern increments as a proportion of total burst depth rather than, say, as an absolute depth in mm, is another example of how regional information can be pooled to represent variability.

4.2.6.2. Dealing with Dependence

It is important to understand whether the variability in one factor might be correlated with another, or whether the nature of variation is dependent upon event magnitude. Again, judgment must be used to determine the appropriateness of data used to investigate such dependencies. If relationships are required on the nature of the dependence between selected hydrologic factors, then evidence can usually be found in meteorologically similar regions. Information on the variability of anthropogenic factors, such as reservoir levels or performance reliability, is also often available from the instrumented record, or from models used to simulate their operations.

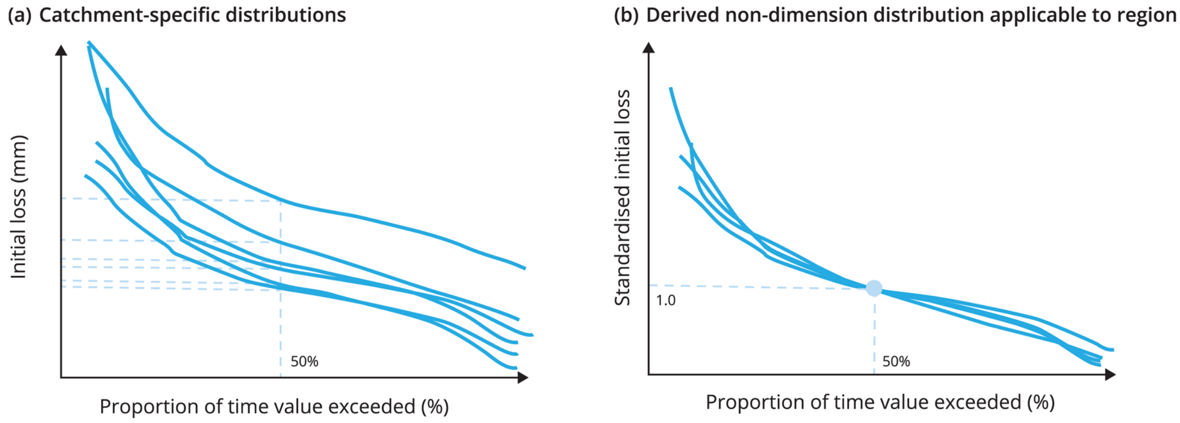

Figure 4.4.4. Use of Standardisation to Derive a Regional Distribution Based on Catchment-specific analyses

Variation with Event Severity

Investigation into how flood producing factors may vary with flood severity may be of particular importance as often the information at the location of interest may be limited. For example, it may be suspected that reservoir levels will be higher at the start of extreme rainfall events as these may be more likely to occur during wetter (La Niña) periods. Evidence for this might be obtained by examining historical correlations between initial reservoir levels prior to large rainfalls, but if such information is limited then it may be more appropriate to "trade space for time" by examining correlations between seasonal rainfalls and extreme storms over a wide region (once the data has been standardised to allow for systematic variation in rainfall depths). An illustration of this by Scorah et al. (2015) for south-eastern Australia is shown in Figure 4.4.5(a).

Two other examples of similar investigations are provided in Figure 4.4.5. The middle panel of Figure 4.4.5 shows the dependence of storm surge on rainfall maxima for an investigation into the interaction between coastal processes and severe weather events (Westra, 2012), and the right-hand panel illustrates the variation in temperature coincident with rainfall maxima for the consideration of the joint probabilities involved in rainfall-on-snow events (Nathan and Bowles, 1997).

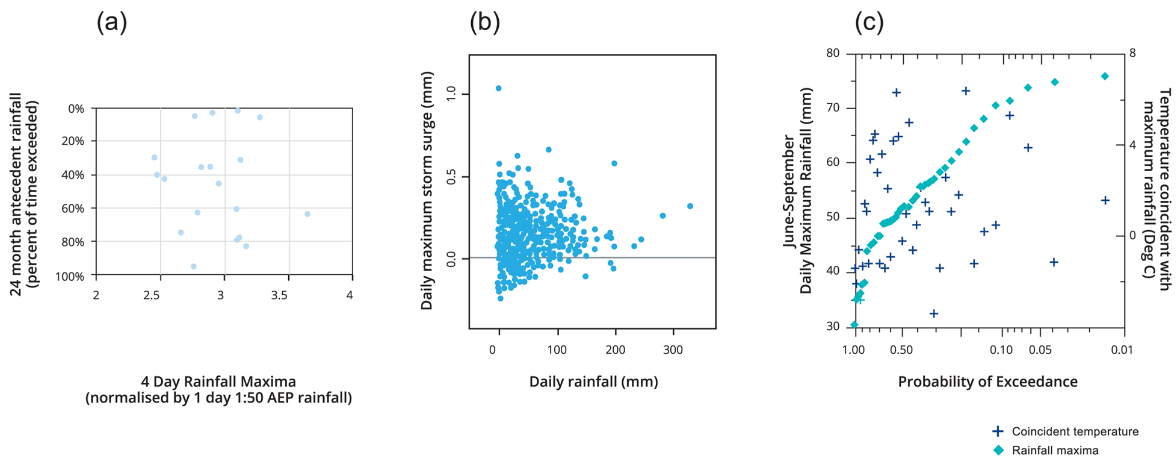

Figure 4.4.5. Examples of Investigations into Dependence between Flood Producing Factors based on (a) Antecedent Seasonal Rainfall Data for Catchments over 1000 km^2 (Scorah et al., 2015), (b) Rainfall and Storm Surge Data (Westra, 2012), (c) and Temperature Coincident with Rainfall Maxima (Nathan and Bowles, 1997)

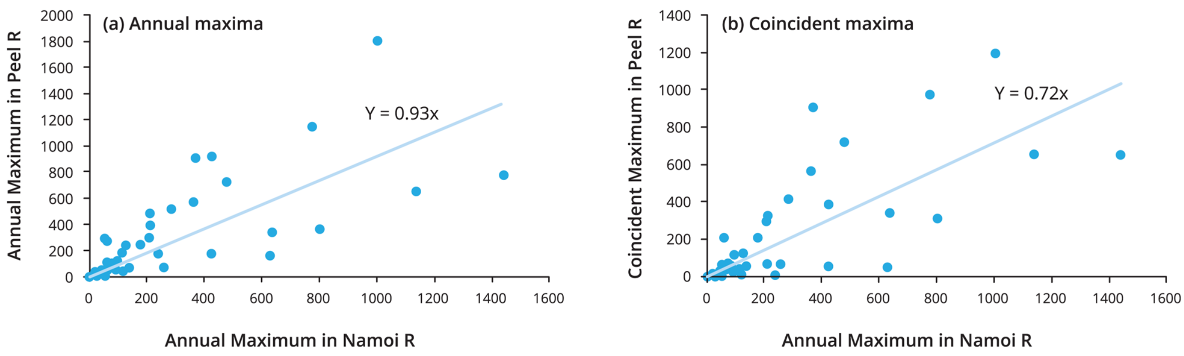

Figure 4.4.6. Examples of Difference in Correlation between Flow Maxima in the Namoi and Peel Rivers, Based on (a) Annual Maxima at Both Sites, and (b) Peel River Flows that are Coincident with Namoi River Maxima

4.2.6.3. Relevance of Sample

Lastly, it is worth stressing the importance of ensuring that the nature of the dependence being investigated is relevant to the design problem. For example, if it is desired to estimate the magnitude of a coincident flood at a downstream confluence to serve as a boundary condition for hydraulic modelling, then the dependency of interest is the flow in the tributary that is coincident with the flow in the mainstream of interest. As shown in Figure 4.4.6, this might well be a different relationship to, say, the correlation between annual maxima at the two sites.

4.3. Monte Carlo Simulation

4.3.1. Introduction

The following sections provide details on some core concepts used in Monte Carlo simulation. The focus of this material is to provide practitioners with sufficient understanding to be able to formulate a scheme that is suited to solving practical problems in flood estimation. A worked example is provided in Book 4, Chapter 4, Section 4 that demonstrates application of the techniques to a practical problem.

A general and very accessible introduction to Monte Carlo methods can be found in Burgman (2005), and more comprehensive and practical guidance is provided in Vose (2000) and Saucier (2000); the latter reference includes C++ source code for a collection of various distributions of random numbers suitable for performing Monte Carlo simulations. Hammersley and Handscomb (1964) provide a more advanced theoretical treatment of the subject, and useful discussion on the advantages of using Monte Carlo methods to estimate design floods can be found in Weinmann et al. (2002), Kuczera et al. (2003), and Weinmann and Nathan (2004).

It should be noted that while there are advantages to developing a simulation framework using high level computing languages such as Python, C++ and Fortran, it is quite feasible to initiate the required design runs and undertake the required statistical analyses using standard spreadsheet software. Robinson et al. (2012) applied such a framework to the solution of the joint probabilities involved in the simulation of extreme floods and reservoir drawdown. At its simplest, any practitioner familiar with the techniques required to prepare batched command scripts and use spreadsheet formulae will be able to implement the procedures described herein.

The following sections outline the main steps involved in developing a Monte Carlo solution of joint probability problems. The sections follow the sequence of steps shown in Figure 4.4.7, which refers to the stochastic deterministic components of the general catchment modelling system as illustrated in Figure 4.4.3. It should also be noted that this scheme is a generalisation of the Monte Carlo framework depicted in Figure 4.3.2 of Book 4, Chapter 3; specifically, the scheme shown in Figure 4.4.7 represents the treatment of natural variability in rainfall-based flood estimation, where no account is given to epistemic uncertainty in the data, parameters, or modelling components.

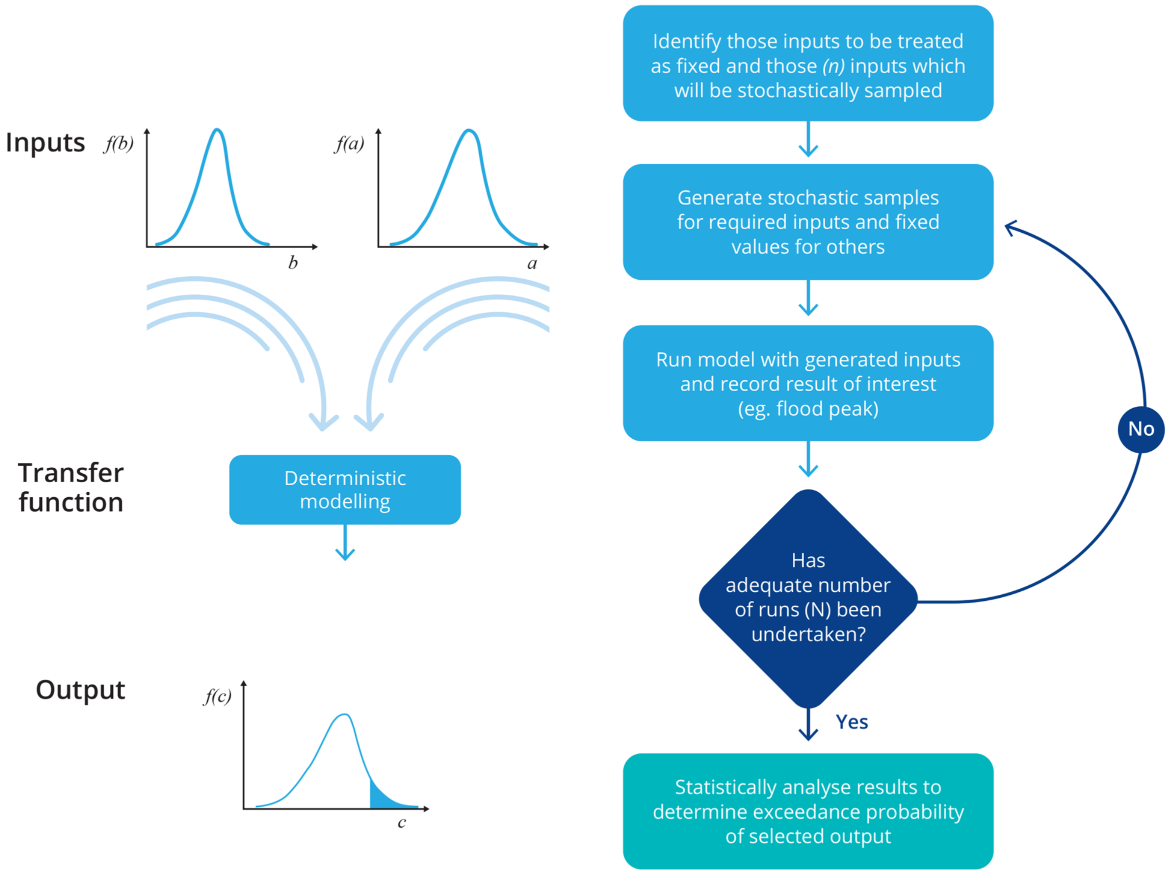

Figure 4.4.7. General Framework for the Analysis of Stochastic Deterministic (Joint Probability) Problems using Monte Carlo Simulation

4.3.2. Generation of Stochastic Inputs

4.3.2.1. Inverse Transformation Approach

The method used to stochastically sample from the input distributions is the core algorithm used in Monte Carlo simulation. Once a suitable framework has been established additional model inputs and/or parameter values can be added to the sampling procedure as required.

The generation scheme makes use of the *inverse transformation* approach. This can be applied to either formally defined probability models, or else to empirical "data-driven" distributions. The basis of the inverse transformation approach is to generate the required probability density function *f(x)* through uniform sampling of the inverse of the cumulative distribution function *F(x)* (ie. the function which gives the probability *P* of *x* being less than a specified value).

The two-step process for doing this is illustrated in Figure 4.4.6, and the algorithm can be summarised as follows:

1. Generate a random number (U) uniformly distributed between 0 and 1;

2. Calculate the value (*x*) of the inverse of the cumulative density function $F^{-1}(U)$.

This process is illustrated in Figure 4.4.8 for three random numbers. The first random number generates a value near the tail of the distribution, and the next two yield values that are more centrally tended. For illustration purposes the input random numbers (*U*) in Figure 4.4.8 are shown as being equally spaced, but on exit the transformed numbers are unequally spaced, in conformance with the adopted distribution. Inverse functions of a number of useful distributions (Normal, log-Normal, Beta, Gamma) are provided in standard spreadsheet software (see example in Book 4, Chapter 4, Section 4). If an empirical distribution is used then values can be simply interpolated from a look-up table comprised of values of the cumulative density function (also see example in Book 4, Chapter 4, Section 4).

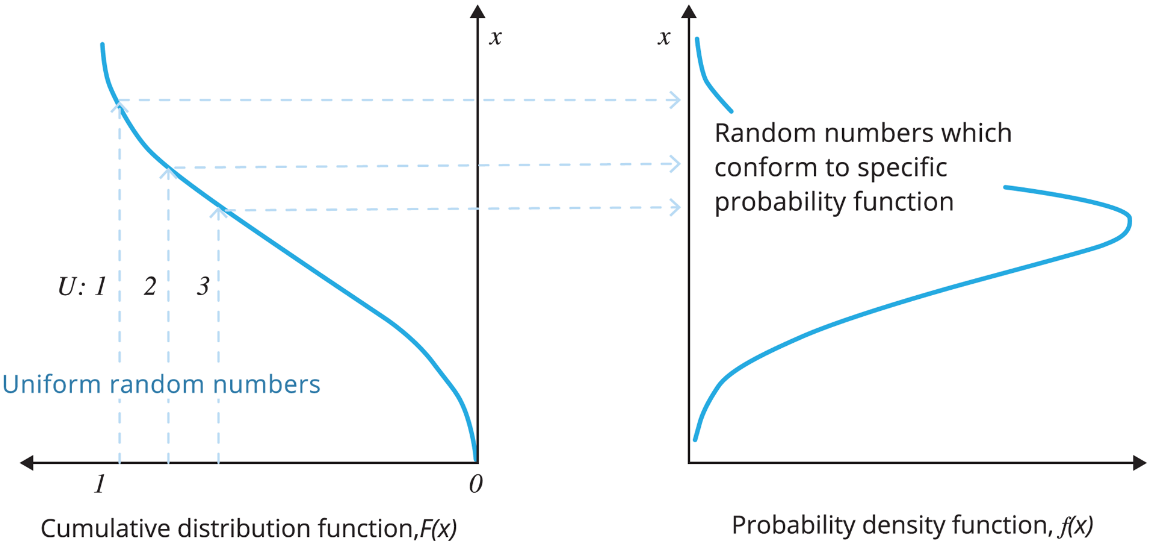

Figure 4.4.8. Inverse Transform Method

4.3.2.2. Parametric Sampling

There are a large number of statistical distributions that can be used to represent variability in different types of hydrological processes and input uncertainty. Information on a range of distributions of potential use can be found in Saucier (2000), Vose (2000), and (Maidment, 1993). Special mention is made here of the Normal distribution. This distribution is also of considerable practical utility as many stochastic processes in hydrology conform to the log-Normal distribution (that is they only take positive values and are skewed towards higher values), and transforming the data beforehand into the logarithmic domain is a simple means of taking direct advantage of the Normal distribution. In addition, many data sets can be transformed into the Normal domain by the Box-Cox transformation (Box and Cox, 1964); with this approach, a variate *X* can be transformed into the Normal domain (*Z*) by the following equations:

$$Z = \frac{X^{\lambda} - 1}{\lambda}, \text{ when } \lambda \neq 0; \; z = \ln(x), \text{ when } \lambda = 0 \qquad (4.4.4)$$

where λ is a parameter determined by trial and error to ensure that the skewness of the transformed distribution is zero. A noteworthy special case of this transformation arises when λ is set to zero, then the transformation is equivalent to taking logarithms of the data. Fitting the parameter λ is most easily achieved by optimisation or the use of “solver” routines that are commonly available in spreadsheet programs. To illustrate the use of the inverse transformation method with a variable that has been transformed using a Box-Cox lambda of 1.2, where the resulting normally-distributed variates have a mean of 50 and a standard deviation of 25:

1. Generate a uniform random number (say, U = 0.548);
2. Derive the value of the inverse cumulative Normal distribution (z = 0.121);
3. Obtain the Normal variate, Z = 50.0+0.121*25 (=53.015);
4. Apply the inverse of the Box-Cox transformation (x = 32.257).

The above four steps can be repeated many hundreds (or thousands) of times as required for input to a model. The outcome of the above four steps repeated 1000 times is provided as a histogram in Figure 4.4.9.

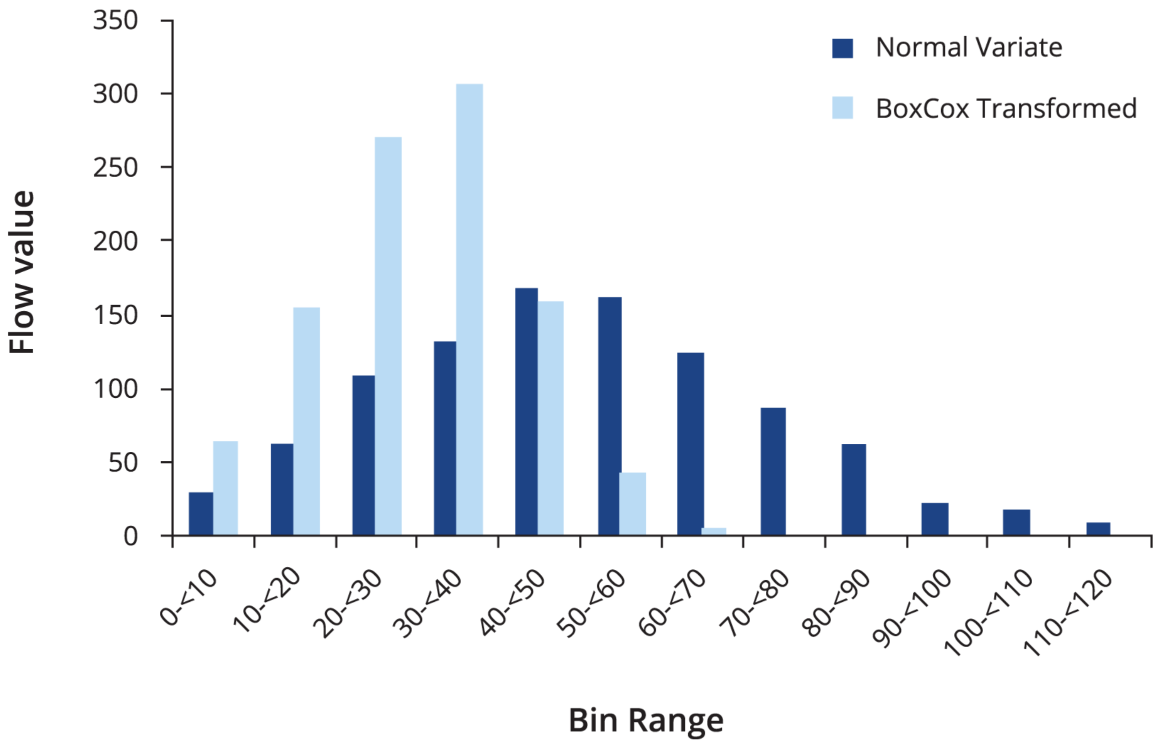

Figure 4.4.9. Histogram Obtained by Generating 1000 Random Numbers Conforming to a Normal Distribution with a Mean of 50 and Standard Deviation of 25, and the Resulting Distribution of variables Obtained by the Inverse Box Cox Transformation (with λ set to 1.20)

Details of the Normal distribution are provided in all statistics textbooks and thus further information will not be presented here. Source code for estimation of the cumulative Normal distribution is freely available (Press et al., 1993) and the function is available in spreadsheet software.

Lastly, it is worth noting that the uniform distribution is also of practical use in flood hydrology. A simple random number generator that varies uniformly between 0 and 1 can be directly applied to the sampling of temporal, or space-time, patterns of rainfall that are considered equally likely to occur.

4.3.2.3. Non-Parametric Sampling

One very practical way of undertaking a Monte Carlo simulation is to sample from a given set of data. This is a fast and simple technique that can be used to take advantage of empirical data sets (such as losses and reservoir drawdown) in a more defensible manner than simple adoption of a single best estimate or representative value. It is also useful for sampling from "pragmatic" distributions, such as rainfall frequency curves that extend beyond 1 in 2000 AEP and which are not based on a theoretical distribution function (Book 2, Chapter 2).

The algorithm to construct and sample from an empirical distribution is as follows:

1. Sort empirical data into either ascending or descending order as appropriate, and assign a cumulative probability value to each. If there are *n* data values, then the largest data value (x_1) is assigned an exceedance probability $F(x_1)$, the second largest (x_2) is assigned an exceedance probability $F(x_2)$, and so on till the last value, represented by x_n and $F(x_n)$;
2. Generate a uniform random number, U = U(0,1);
3. Locate interval *i* such that $F\left(x_i\right) \leq U < F\left(x_{i+1}\right)$;
4. Return $X = x_i + \frac{U - F\left(x_i\right)}{F\left(x_{i+1}\right) - F\left(x_i\right)}\left(x_{i-1} - x_i\right)$;
5. Generate additional points by returning to Step 2.

While simple to implement, the use of empirical distributions in Monte Carlo simulation does require care. Most importantly, it is necessary to ensure that the data sample being used is relevant to the whole range of conditions being simulated. For example, if the data set is comprised of initial reservoir levels recorded over a short historic period, then these may not be relevant to the assessment of extreme flood risks under a different set of operating rules.

It is seen in Step 4 of the above algorithm that values within each interval are obtained by linear interpolation. This is normally quite acceptable, though obviously the less linear the relationship between the data values and their corresponding exceedance probabilities the less defensible is such an approach. Accordingly, in some cases it is best to first transform the data and/or the exceedance probabilities assembled for Sstep 1 of the algorithm. Many hydrological variables are approximately log-Normally distributed, and thus it is often desirable to undertake the interpolation in the log-Normal domain. To this end, the ranked data values are transformed into logarithms (it does not matter what base is used) and the exceedance probabilities are converted to a standard normal variate (that is, the inverse of the standard normal cumulative distribution). Step 2 of the above algorithm would thus need to be replaced with $U = U(z_{min}, z_{max})$ where $z_{min}$and z_{max} represent the standard normal deviates corresponding to $F(x_1)$ and $F(x_n)$, ie. the adopted limits of exceedance probability range.

Care is also required when sampling from the tails of the distribution. Empirical data sets are of finite size and, if the generated data are to fall between the upper and lower limits of the observed data, the cumulative exceedance probability of the first ranked value $F(x_1)$ should be zero, and that of the last ranked value $F(x_1)$ should be 1.0. Thus use of empirical data sets is appropriate for those inputs whose extremes of behaviour are not of great relevance to the output. Losses, for example, are zero bounded, and thus the difference in flood peak between a loss exceeded 95% of the time and that exceeded 99.999% of the time may well be of no practical significance. However, if an empirical approach is being used for the generation of rainfalls that are defined for between 1 in 2 and 1 in 100 AEP, then it is inevitable that more than half the random numbers generated in Step 2 of the above algorithm can be expected to lie outside the specified range of rainfalls. As long as the probability range of interest lies well within the limits specified, then rainfall values can be obtained by some form of appropriate extrapolation; however, if this approach is used then checks should be undertaken to ensure that the extrapolated values do not influence the results of interest.

4.3.2.4. Generating Correlated Variables

Many hydrologic variables are correlated and thus it is sometimes necessary to ensure that the adopted sampling scheme preserves the correlation structure of the inputs. A simple means of generating correlated variables is described by Saucier (2000). The approach is based on rotational transformation and the steps involved in generation of uniformly distributed variates can be stated as follows:

1. Independently generate two uniform random variates, *X= U(-1,1)* and *Z= U(-1,1)*;
2. Set $Y = \rho X + Z\sqrt{1-\rho^2}$ where *r* is the required correlation between *X* and *Z*;
3. Return:

$$x = \frac{x_{\min} + x_{\max}}{2} + X\left(\frac{x_{\max} - x_{\min}}{2}\right)$$

$$y = \frac{y_{\min} + y_{\max}}{2} + Y\left(\frac{y_{\max} - y_{\min}}{2}\right)$$

   where x_{min} and x_{max} are the lower and upper bounds of the first variate and y_{min} and y_{max} are the corresponding bounds of the other.

Application of the above algorithm is illustrated in Figure 4.4.10(a). The bounds along the x-axis are 5 and 130, and those along the y-axis (for the mid-point of the x distribution) are 30 and 75. Figure 4.4.10 illustrates the results for the generation of 2000 correlated variates where the correlation coefficient (ρ) adopted is -0.7.

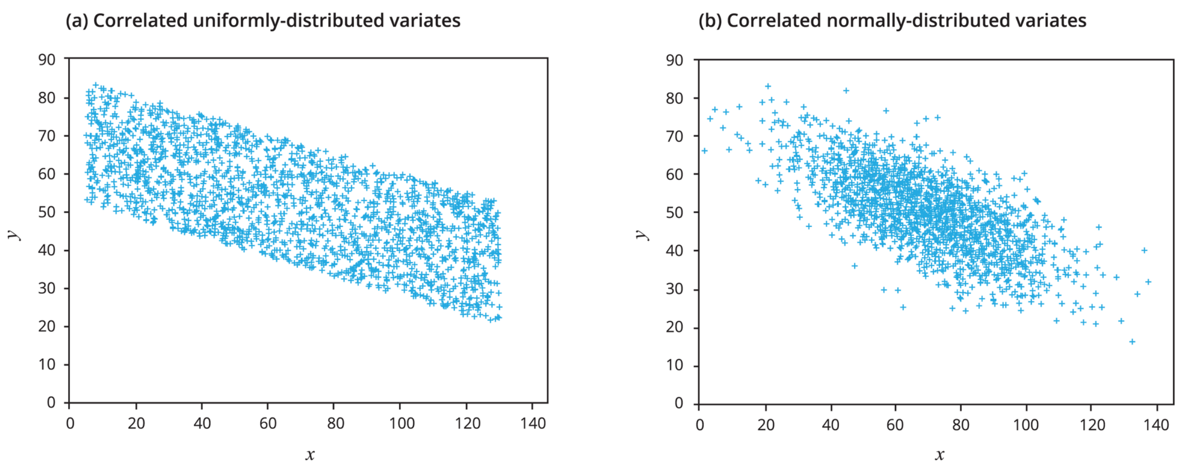

Figure 4.4.10. Generation of Variables with a Correlation of -0.7 based on (a) Uniform and (b) Normal Distributions

The above algorithm can easily be adapted to the generation of correlated variates that conform to some specified distribution. For the Normal distribution, the required algorithm is:

1. Independently generate two normal random variates with a mean of zero and a standard deviation of 1: *X= N(0,1) and Z= N(0,1)*;

2. Set $Y = \rho X + Z\sqrt{1-\rho^2}$ where *r* is the required correlation between *X* and *Z*;

3. Return:

   $x = \mu_x + X\sigma_x$

   $y = \mu_y + Y\sigma_y$

   where μ_x and μ_y are the means of the two distributions and σ_x and σ_y are the required standard deviations.

Application of the above algorithm is illustrated in Figure 4.4.10(b). The input parameters to this example are $\rho = -0.7$, $\mu_x = 70$ and $\sigma_x = 10$, and $\mu_y = 50$ and $\sigma_y = 10$ and as before a total of 2000 correlated variates are generated. Any distribution could be used in lieu of the Normal distribution, or else the variates of interest could be transformed into the normal domain.

4.3.2.5. Conditional Sampling

The preceding two sections provide a means for generating "well-behaved" variables that can be fitted to a suitable function or distribution. However, many correlated hydrologic variables are awkwardly distributed and their variability is dependent on some (often non-linear) function of their magnitude. A typical example of this type of correlation is the manner in which the level in an upstream reservoir is weakly dependent on the level in a downstream reservoir. The nature of one such dependence is shown by the large solid symbols in Figure 4.4.11, which is derived from the behaviour of two reservoirs located in south-eastern Australia. Such data is difficult to normalise or fit to probability distributions, and thus an empirical sampling approach can be used.

The approach that can be followed to stochastically sample from such a data set can be described as follows:

1. Identify the "primary" variable that is most important to the problem of interest, and prepare a scatter plot of the two variables with the primary variable plotted on the x-axis (as shown in Figure 4.4.11);

2. Divide the primary variable into a number of ranges such that variation of the dependent variable (plotted on the y-axis) within each range is reasonably similar; in the example shown in Figure 4.4.11 a total of seven intervals has been adopted as being adequate. This provides samples of the secondary variable that are conditional on the value of the primary variable;

3. Stochastically generate data for the primary variable using the empirical approach as described in Book 4, Chapter 4, Section 3;

4. Derive an empirical distribution of the dependent data for each of the conditional samples identified in Step 2 above (that is, undertake Step 1 of the empirical approach as described in Book 4, Chapter 4, Section 3 for each of the intervals); thus, for the example shown in Figure 4.4.11 a total of seven separate empirical distributions of upstream storage levels are prepared;

5. For each generated value of the primary variable, stochastically sample from the conditional distribution corresponding to the interval that it falls within; for example, if a downstream storage level of 1500 ML was generated in Step 3 above, then the empirical

approach described in Book 4, Chapter 4, Section 3 is applied to the conditional distribution obtained from data occurring within the third lowest interval shown in Figure 4.4.11.

The results from application of the above procedure are illustrated in Figure 4.4.11 for 2000 stochastic samples (shown by the blue “+” symbols). The 2000 correlated values are stochastically generated based on information contained in 500 observations. It is seen that the correlation structure in the observed data set is preserved reasonably well by this procedure.

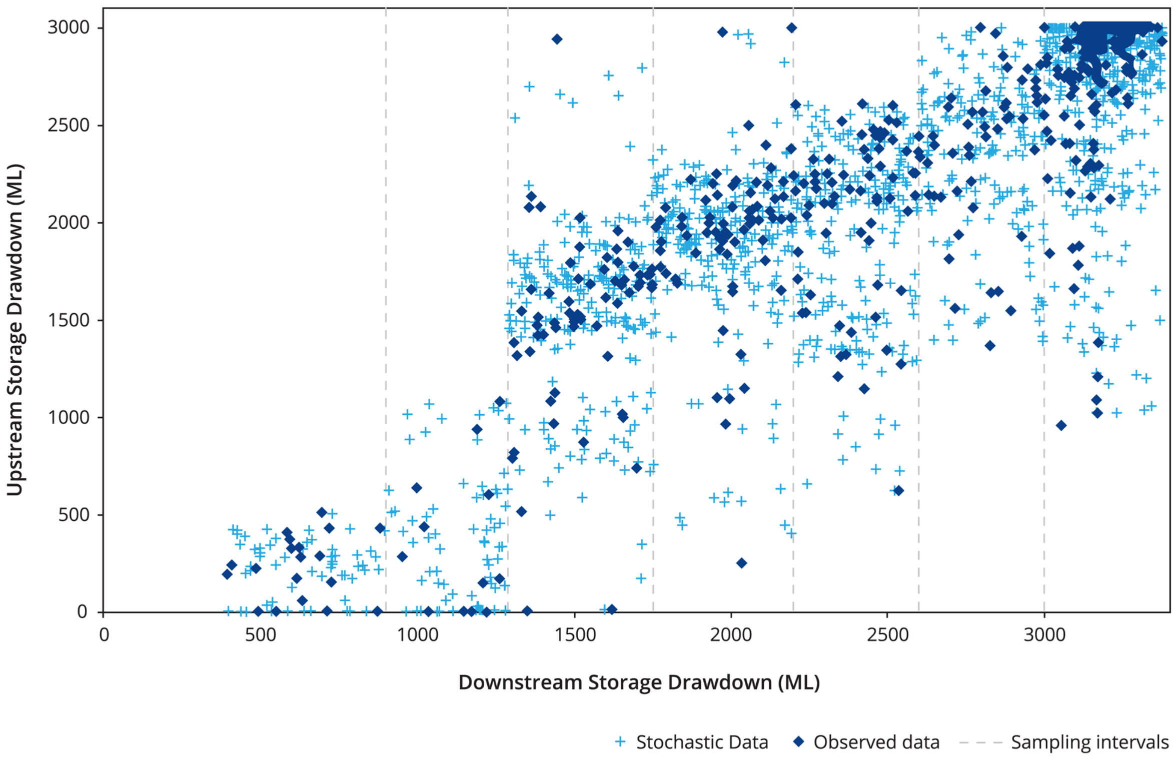

Figure 4.4.11. Conditional Empirical Sampling - Storage Volume in an Upstream Dam is Correlated with the Volume in a Downstream Dam

4.3.3. Estimation of Exceedance Probabilities

4.3.3.1. Selection of Method

Estimation of exceedance probabilities from Monte Carlo simulation results can be obtained by either “direct sampling” or “stratified sampling” approaches. With direct sampling, the results are analysed using either traditional frequency analysis or non-parametric methods; with stratified sampling, the results are analysed by application of the Total Probability Theorem. The decision regarding which approach to use is largely a practical one, though there are theoretical differences in the nature of the derived quantiles: application of the Total Probability Theorem yields *expected probability estimates* of a given flood magnitude, whereas traditional frequency analysis of the derived maxima based on Cunnane (and most other) plotting positions are formulated to yield *unbiased estimates of the flood magnitude* for a given exceedance probability, though adoption of the Weibull plotting position $i/(n+1)$ should yield unbiased probability estimates (Book 3, Chapter 2, Section 6). It is always necessary to experiment with many different model parameters, model configurations, and design scenarios, and simulation times of more than an hour or so soon become impractical.

The first approach, based on direct sampling, is the most straightforward to implement. It is well suited to the analysis of problems that can be computed quickly, or else to more complex problems in which the probability range of interest is limited to reasonably frequent events. As a rule of thumb, the number of simulations required is around 10 to 100 times the largest average recurrence interval of interest. That is, if the rarest event of interest has an annual exceedance probability of 0.001, then it will be necessary to generate between 10 000 to 100 000 stochastic samples in order to derive a stable result.

The second approach, based on stratified sampling, does require more effort to implement. It can still be formulated using a “batch” file approach, though additional care needs to be taken with how the inputs are formulated and the results analysed. The benefit of this effort is that the number of runs required to estimate the exceedance probability of rare events is considerably fewer; indeed the algorithm can be designed so that a similar number of runs is required regardless of the range of probabilities of interest.

Further information on these two approaches is provided in the next two sections. It is worth noting that other approaches could be used; for example Diermanse et al. (2014) derive estimates using importance sampling, which is similarly efficient to the stratified sampling discussed below.

4.3.3.2. Direct Sampling

The results output from the Monte Carlo simulation are most easily analysed by non-parametric frequency analysis. Using flood peaks as an illustration, the steps involved can be summarised as follows:

1. Sort the *N* simulated peaks in order of decreasing magnitude;

2. Assign a rank (i) to each peak value; 1 to the highest value, 2 to the next highest, and so on, down to rank *N*;

3. Calculate the plotting position (*p*) of each ranked value using either the Weibull (Equation (4.4.5)) or the Cunnane (Equation (4.4.6)) formulae:

$$p = \frac{i}{N+1} \tag{4.4.5}$$

$$p = \frac{i-0.4}{N+0.2} \tag{4.4.6}$$

   If the design focus is on estimating the *probability* of a given flood magnitude then the Weibull formula (Equation (4.4.5)) should be used as this provides an unbiased estimate of the exceedance probability of any distribution. Alternatively, if the focus is on the *magnitude* associated with a given exceedance probability then the Cunnane formula (Equation (4.4.6)) is preferred as this provides approximately unbiased quantiles for a range of distributions.

4. Construct a probability plot of the ranked peaks against their corresponding plotting positions. The plot scales should be chosen so that the frequency curve defined by the plotted values is as linear as possible. In many hydrological applications the ranked values may be plotted on arithmetic or log scales and the estimated exceedance probabilities (the plotting positions) are plotted on a suitable probability scale. Most popular spreadsheet programs do not include probability scales and thus, for probability plots conforming approximately to the Normal or log-Normal distribution, it is necessary to

convert the probabilities to their corresponding standard normal cumulative distribution values. Alternatively, for probability plots conforming approximately to the exponential distribution, the reciprocal of the exceedance probabilities (the average recurrence interval) can be plotted on a logarithmic scale; and

5. The magnitude associated with a given exceedance probability (if the Cunnane plotting position is used) or else the exceedance probability associated with a given magnitude (if the Weibull plotting position is used) can be interpolated directly from the probability plot. For convenience, a suitable smoothing function (ie. polynomial equation) can be fitted to the plotted values in the region of interest to simplify the estimation of design values. The function is used merely to interpolate within the body of the plotted points and thus, as long as there is no bias in the fit, it matters little what function is used (polynomial functions are quite suitable).

If desired, the maxima can be fitted using a traditional probability model (Book 3, Chapter 2), but given that sufficient simulations need to be undertaken to yield a stable estimate, there is little point in doing so.

4.3.3.3. Stratified Sampling

While the above approach is straightforward, it is computationally inefficient as the vast majority of simulations undertaken provide little information on the extremes of interest. That is, the vast majority of computational effort is expended on deriving results for the range of exceedance probabilities that is of least interest. This inefficiency is of little concern when using simple models with sparing outputs and fast simulation speeds. However, as the data processing becomes more complicated and execution speeds increase, simulation times and data storage requirements quickly pose significant practical problems.

Adoption of a stratified sampling approach ensures that the computational effort is always focused on the region of interest and, if the simulation scheme is configured carefully, then it will usually be possible to apply Monte Carlo simulation to most practical problems.

The approach follows the same logic as represented in the flow chart of Figure 4.4.7, the only difference is that samples of the stochastic variable that is of most importance to the output are generated over specific probability ranges. It matters little how the ranges are defined and the ranges can be varied to suit the different ranges of interest. It is simplest to divide the domain into *M* intervals uniformly spaced over the standardised normal probability domain (Detail A in Figure 4.4.12). It should be noted that adopting this approach does not make any distributional assumption about the variable, it simply provides the means to distribute the simulations evenly across the probability domain. Typically 50 intervals should suffice, though care is required to ensure that there is adequate sampling over the region of most interest.

In the example illustrated in Figure 4.4.12, rainfall is used as the primary stochastic variable. Within each interval *N* rainfall depths are stochastically sampled and for each rainfall depth a model simulation is undertaken using an appropriate set of stochastic inputs (Detail B in Figure 4.4.12). The number of simulations specified in each interval (*N*) is dependent on the number of inputs being stochastically generated and their degree of variability, but in general it would be expected that between 50 and 200 simulations should be sufficient to adequately sample from the range of associated inputs.

The model results are recorded for all simulations taken in each interval (Detail C in Figure 4.4.12). These results are assessed using the Total Probability Theorem (Book 4, Chapter 4, Section 2) to yield expected probability estimates of the flood frequency curve. In all, if the rainfall frequency curve is divided into 50 intervals and 200 simulations are undertaken in each interval, a total of 10 000 runs is required. The same number of simulations could be used whether the upper limit of exceedance probability is 1 in 100 or 1 in 10^6, and it is merely necessary to ensure that a representative number of combinations is sampled within each rainfall range of interest. If the distribution of different rainfall durations is known, the Total Probability Theorem can also be used to give appropriate weighting to separate flood simulations for different rainfall duration intervals.

For the scheme illustrated in Figure 4.4.12, the expected probability that a flood peak (Q) exceeds a particular value q can be calculated from the Total Probability Theorem:

$$p(Q > q) = \sum_i p[Q > q|R_i]p[R_i] \tag{4.4.7}$$

where the term $p[R_i]$ represents the probability that rainfall occurs within the interval i, and the term $p[Q > R_i]$ denotes the conditional probability that the flood peak Q generated using a rainfall depth from within this interval R_i exceeds q. The term $p[R_i]$ is simply the width of the probability interval under consideration (this will be different for each of the M intervals considered), and $p[Q > R_i]$ can be calculated merely as the proportion of exceedances, n, in the sample of N simulations within interval i (ie. as n/N). A representative value of R can be used for all N simulations within the interval, though a smoother frequency curve can be obtained if R is sampled with the interval using a uniform distribution.

In order to ensure that the total probability domain is sampled, it is necessary to treat the first and last intervals differently from the intermediate ones. The issue here is that the full extents of the end intervals have to be adequately sampled, and on the assumption that these boundary intervals are distant from the probability region of interest, we can estimate their contribution to the total probability in a pragmatic fashion. For the last interval $p[R_1]$ is evaluated as the exceedance probability of its lower bound, and for the first interval it is evaluated as the non-exceedance probability of its upper bound. Also, for the first interval $p[Q > q|R_1]$ is replaced by the geometric mean of $p[Q > q|R_1*]$ and, say, 0.1 x$p[Q > q|R_1*]$, where R_1* is the rainfall value at the upper bound of the interval. Similarly, for the last interval the term $p[Q > q|R_\mathrm{N}]$ is replaced by the geometric mean of $p[Q > q|R_\mathrm{N}*]$ and 1.0, where R_N* is the rainfall value at the lower bound of the interval. Thus, we are assuming for the lowest interval that as the frequency of the rainfall event becomes very high the likelihood that the flow threshold is exceeded trends towards a very low value, in this case taken as one tenth the probability of $p[Q > q|R_1*]$; and for the uppermost interval we assume that the likelihood of the threshold being exceeded trends towards a value of 1.0 (ie. a certainty). The geometric mean is used in place of the arithmetic mean as here we are assuming a highly non-linear variation over the interval.

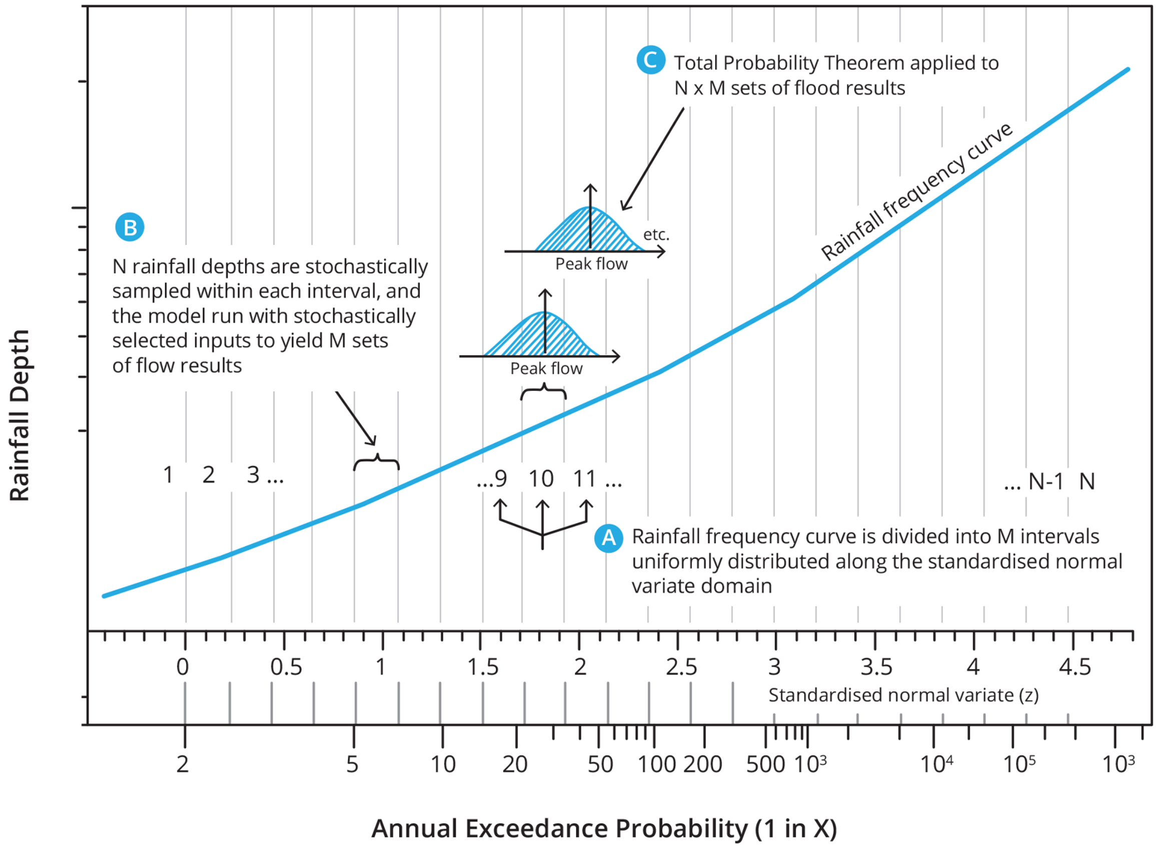

Figure 4.4.12. Manner in which Stratified Sampling is Applied to the Rainfall Frequency Curve

4.4. Example

The example below shows how the concepts described in this chapter may be used to solve a commonly encountered practical problem. The example is based on real data, but has been adapted somewhat to more easily illustrate the concepts involved.

The case study involves a township that is located below the confluence of two rivers (Figure 4.4.13). Both rivers are gauged, and one (referred to here as the “mainstream”) is larger than the other (the “tributary”). Flood frequency information has been derived for the two gauging sites, and the main focus of the study is to derive 1% AEP flood levels below the confluence, immediately upstream of the town. A one dimensional (HEC-RAS) model has been developed for the valley to allow flood levels to be determined throughout the town. The portion of the model of most relevance to this problem is shown by blueshading in Figure 4.4.13.

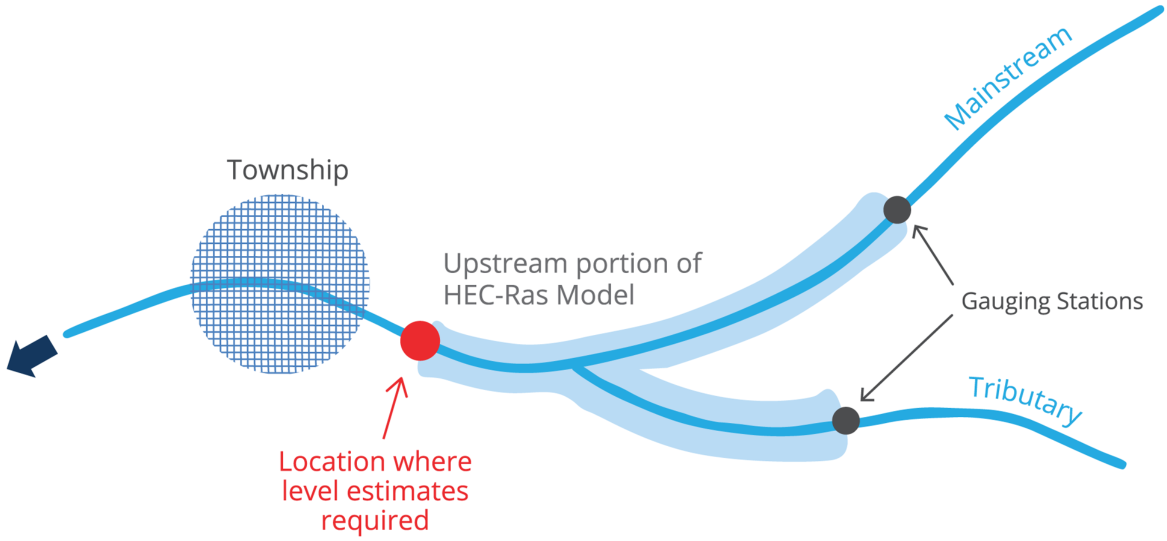

Figure 4.4.13. Schematic layout of example joint probability problem.

The analysis of this problem follows the components as outlined in Figure 4.4.7. Flood levels upstream of the town may be the result of a large flood in the mainstream with a small tributary flood, or a large flood in the tributary with average flow conditions in the mainstream; more commonly, it might be expected that the downstream levels are a function of different extremes of flooding in both contributing rivers. Flood Frequency Analysis was undertaken on the Annual Maxima Series derived at both gauges, and it was found that a log-Normal distribution provided an adequate fit to both (Figure 4.4.14a). An analysis of the coincident flow maxima at both sites indicated that the correlation between flood peaks was 0.6, and a scatter plot of the historic peaks used to make this inference is shown in Figure 4.4.14 b).

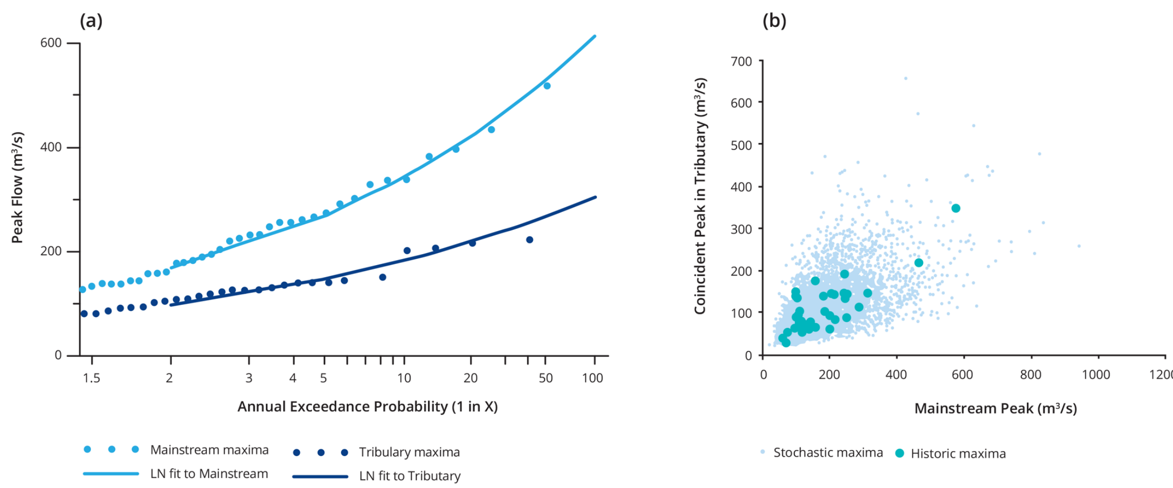

Figure 4.4.14. (a) Flood Frequency Curves for the Mainstream and Tributary gauging sites, and (b) Correlation between Historic Flood Peaks and Sample of Generated Maxima

The first step in the process is to generate the correlated stochastic inputs relevant to the two branches of the stream. This is done using the procedure outlined in Book 4, Chapter 4, Section 3 in conjunction with the inverse transform method (Book 4, Chapter 4, Section 3). The first ten rows of the simulation are shown in Table 4.4.1. Uniform random numbers are provided in Columns 2 and 3, and Columns 3 and 4 show the corresponding values of the inverse cumulative Normal distribution (the standard normal variates). Column 6 shows the correlated value of the standard normal variate, which is obtained from the procedure outlined in Book 4, Chapter 4, Section 3; however, as here a orrelated standard normal variate is generated rather than a correlated uniform variates, the two input variables are *X= N(0,1)* and *Z= N(0,1)*, ie. Columns 4 and 5, not columns 1 and 2. The corresponding maxima in the mainstream and the tributary are shown in Columns 7 and 8, and are obtained by scaling the N(0,1) variates by the relevant means and standard deviation of the log-Normal distribution, e.g. $x = \mu_x + X\sigma_x$ The mean and standard deviation for both streams are shown at the top of the table in Columns 4 and 5, and the results shown in Columns 7 and 8 have been transformed back into the arithmetic domain by taking the anti-log of *x*. The results of applying these steps 5000 times are shown in Figure 4.4.14(b).

Table 4.4.1. Stochastic Generation of Correlated log-Normal Maxima

			Mainstream	Tributary			Intercept	8.06727
		Mean	2.2146	1.9975			a	0.00402
		Std Deviation	0.2194	0.2228			b	0.00156
		Correlation	0.6				N	5000
Column 1	Column 2	Column 3	Column 4	Column 5	Column 6	Column 7	Column 8	Column 9
Count	U_x	U_y	X	Z	Y	Mainstream (m^3/s)	Tributary (m^3/s)	Level (m)
1	0.0608	0.3890	-1.5478	-0.2820	-1.1543	75.0	55.0	8.455
2	0.3928	0.3538	-0.2719	-0.3752	-0.4633	142.9	78.4	8.765
3	0.6415	0.3207	0.3625	-0.4659	-0.1552	196.9	91.4	9.003
4	0.1871	0.9256	-0.8887	1.4438	0.6218	104.6	136.8	8.702
5	0.5970	0.4625	0.2457	-0.0941	0.0722	185.6	103.2	8.975
6	0.6556	0.0662	0.4005	-1.5045	-0.9633	200.7	60.7	8.970
7	0.3334	0.1897	-0.4304	-0.8789	-0.9614	131.9	60.7	8.693
8	0.9805	0.6330	2.0647	0.3399	1.5107	465.2	215.8	10.277
9	0.1692	0.3399	-0.9572	-0.4128	-0.9045	101.1	62.5	8.572
10	0.2268	0.2388	-0.7494	-0.7100	-1.0177	112.3	59.0	8.611

The next step in the process is to derive the deterministic component of the system. To this end, representative flows were input into a HEC-RAS model of the stream and the results levels were obtained. Seven pairs of simulations were undertaken as shown in Figure 4.4.15 and Table 4.4.2. A multiple regression model was fitted to this information, and the resulting relationship is depicted in Figure 4.4.15. This function is used in Column 9 of Table 4.4.1 to obtain the flood level resulting from the stochastic maxima provided in Columns 7 and 8.

A probability plot of the ranked 5000 stochastic flood levels (using the Weibull plotting position formula) is depicted in Figure 4.4.16. The 1% AEP flood level may be found by simple linear interpolation of these results, and is found to be a level of 10.55 m. Also shown in Figure 4.4.16 is the dependence of this estimate on the degree of correlation between the mainstream and tributary peaks, where it is seen that if the peaks are assumed to be fully independent or dependent the flood level estimate varies between 10.40 and 10.73 m, respectively.

It is worth noting that trials were undertaken to determine how many simulations were required to yield stable estimates of the quantiles. In this example, there was no difference in results if 1000 or 5000 simulations were used, though below this number the estimates started to become unstable.

Table 4.4.2. Derivation of Deterministic Function Relating Upstream Flows to Downstream Levels (a)

Peak in Mainstream (m^3/s)	Peak in Tributary (m^3/s)	Flood Level (m)
248.1	286.0	9.54
320.0	283.2	9.75
393.6	274.1	10.05
424.8	260.8	10.22
444.6	242.1	10.33
458.7	196.0	10.12
464.4	0.1	9.95

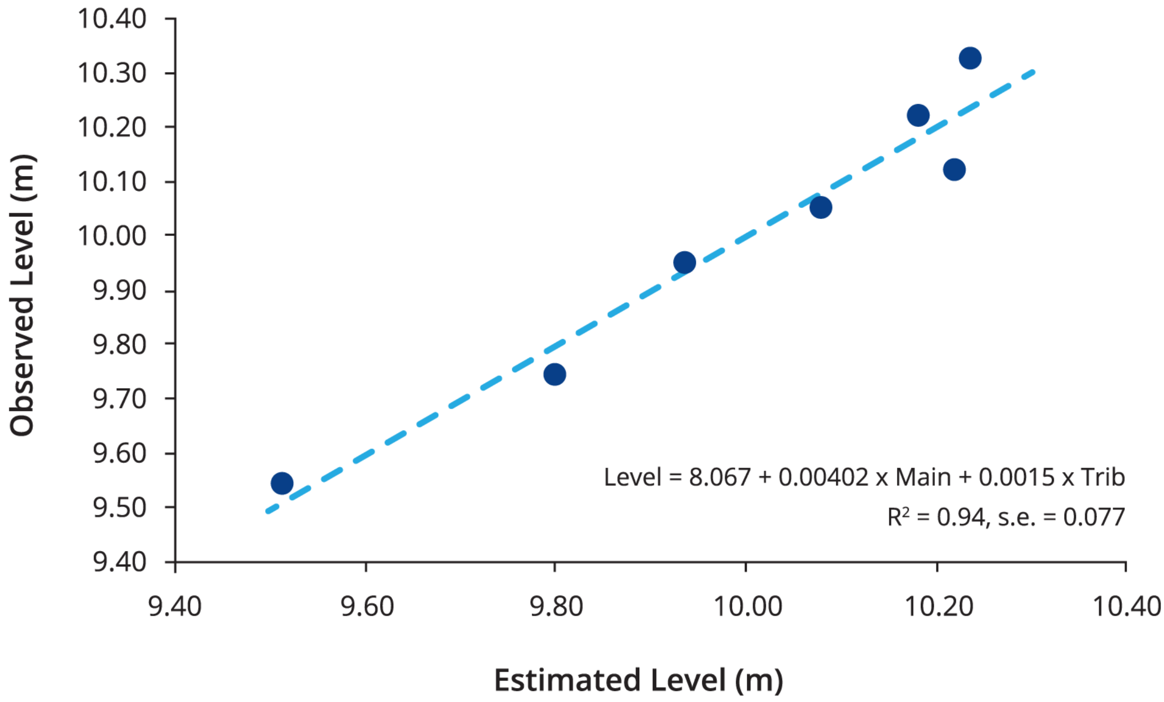

Figure 4.4.15. Derivation of Deterministic Function Relating Upstream Flows to Downstream Levels

Lastly, an estimate of the exceedance probability can be obtained using stratified sampling and use of the Total Probability Theorem. To this end, the probability domain was divided into 10 divisions, and 20 simulations were undertaken in each (totalling 200 simulations). The boundaries of the ten divisions are shown in Columns 2 and 3 of Table 4.4.3, where the limits have been uniformly distributed between standard normal variates of 1 and 4. The calculations are undertaken as described in Book 4, Chapter 4, Section 3 for the level threshold of 10.4 m, where the conditional probability terms are based on the exceedance probability of flows in the mainstream. The probability of an event occurring in each of the ten bins is shown in Column 4, and this is determined from the exceedance probabilities associated with each of the bins. For example, the probability that a flow in the mainstream lies within the first bin is simply the difference between 0.90320 and 0.84134 (= 0.06185), which are the probabilities of the normal distribution that correspond to the standard normal variates of 1.00 and 1.30. The number of times that a level exceeds 10.4 m in each bin is given in Column 5, and the corresponding conditional probability is shown in Column 6, which is computed by dividing by the number of samples in each bin (which in this case is 20). The product of the conditional probability term (Column 6) and the interval width (Column 4) is given in Column 7, and the summation is provided at the bottom of the table. It is thus seen that the exceedance probability of exceeding 10.4 m is estimated to be 0.0149 (or around 1 in 70). A comparison between three such estimates and the results obtained from simple simulation is shown in Figure 4.4.16, from which is seen that the results obtained are similar.

Table 4.4.3. Calculation of Exceedance Probability of the Level Exceeding 10.4 m using the Total Probability Theorem

Column 1	Column 2	Column 3	Column 4	Column 5	Column 6	Column 7
Bin	Z_{min}	Z_{max}	$p[M_i]$	Num [H>h]	$p[H>h\|M_i]$	$p[H>h\|M_i]*p[H>h]$
1	1.00	1.30	0.061855	0	0.00	0.000000
2	1.30	1.60	0.042001	0	0.00	0.000000
3	1.60	1.90	0.026083	0	0.00	0.000000
4	1.90	2.20	0.014813	4	0.20	0.002963
5	2.20	2.50	0.007694	15	0.75	0.005770
6	2.50	2.80	0.003655	20	1.00	0.003655
7	2.80	3.10	0.001588	20	1.00	0.001588
8	3.10	3.40	0.000631	20	1.00	0.000631
9	3.40	3.70	0.000229	20	1.00	0.000229
10	3.70	4.00	0.000076	20	1.00	0.000076
						0.014911

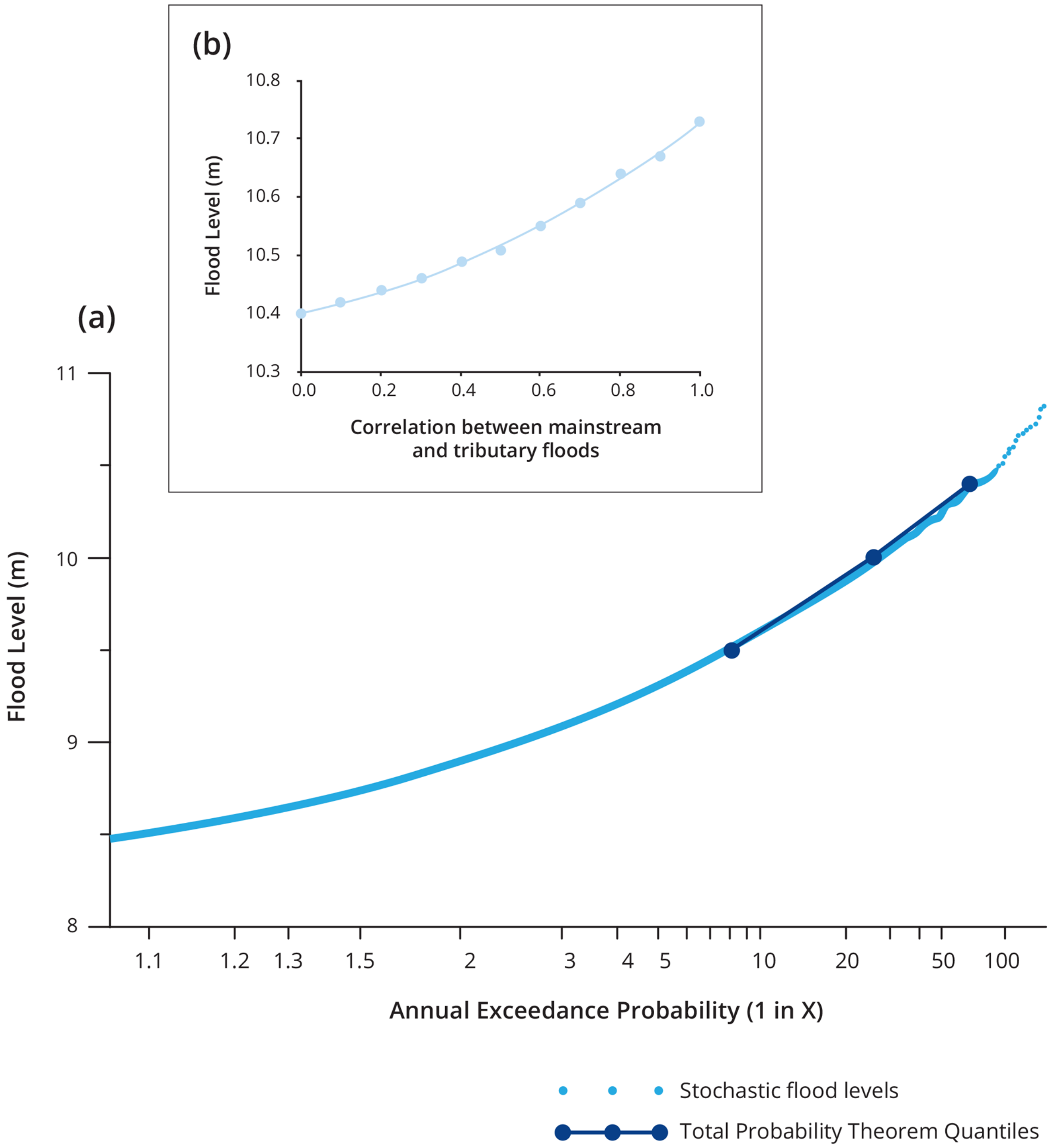

Figure 4.4.16. Derived Frequency Curve of Downstream Levels, with (b) Dependence of 1% Annual Exceedance Probability Level on Degree of Correlation between Flood Peaks

4.5. References

Benjamin, J.R. and Cornell, C.A. (1970), Probability, Statistics and Decisions for Civil Engineers. McGraw-Hill, New York.

Box, G.E.P. and Cox, D.R. (1964), An Analysis of Transformations, J Royal. Statistical Society, 211-243, discussion 244-252.

Burgman, M. (2005), Risks and Decisions for Conservation and Environmental Management. Cambridge University Press.

Chen, L., Singh, V.P., Shenglian, G., Hao, Z. and Li, T. (2012), Flood Coincidence Risk Analysis Using Multivariate Copula Functions, J. Hydrol. Eng., pp: 742-755, Diermanse, F.L.M., Carroll, D.G., Beckers, J.V.L., Ayre, R. and Schuurmans, J.M. (2014), A Monte Carlo framework for the Brisbane river catchment flood study. Hydrology and Water Resources Symposium 2014. Barton, ACT: Engineers Australia.

Favre, A.C., El Adlouni, S., Perreault, L., Thiémonge, N. and Bobée, B. (2004), Multivariate hydrological frequency analysis using copulas. Water Resour. Res., 40(1).

Fricke, T.J., Kennedy, M.R. and Wellington, N.B. (1983), The Use of Rainfall Correlation in Determining Design Storms for Waterways on a Long Railway Line. In: Hydrology and Water Resources Symposium (15th : 1983 : Hobart, Tas.). National conference publication (Institution of Engineers, Australia), (83/13), 215-219.

Genest, C., and Favre, A. (2007), Everything You Always Wanted to Know about Copula Modeling but Were Afraid to Ask. Journal of Hydrologic Engineering, 12(4), 347-368.

Haan, C.T. (1974), Statistical Methods in Hydrology, Second Edition. Iowa State Press.

Hammersley, J.M. and Handscomb, D.C. (1964), Monte Carlo Methods, Methuen and Co, London; John Wiley and Sone, New York.

Hill, P.I., Graszkiewicz, S., Loveridge, M., Nathan, R. and Scorah, M. (2015), Analysis of loss values for Australian rural catchments to underpin ARR guidance. Proc. 36th Hydrology and Water Resources Symposium, Hobart.

Jordan, P., Nathan, R., Weeks, B., Waskiw, P., Heron, A., Cetin, L., Rogencamp, G., Stephens, C. and Russell, C. (2015), Estimation of flood risk for linear transport infrastructure using continuous simulation modelling. Proc. 36th Hydrology and Water Resources Symposium, Hobart.

Kilgore, R.T., Thompson, D.B. and Ford, D.T. (2010), Estimating Joint Probabilities of Design Coincident Flows at Stream Confluences. National Cooperative Highway Research Program Report 15-36, Washington D.C.

Kuczera, G., Lambert, M., Heneker, T., Jennings, S., Frost, A., Coombes, P. (2003), Joint probability and design storms at the crossroads. Proc, 28th Int. Hydrology and Water Resour. Symp. 10-14 Nov, Wollongong.

Laurenson, E.M. (1974), Modeling of stochastic-deterministic hydrologic systems. Water Resources Research, 10(1), 955-961.

Maidment, D.R. (Ed) (1993). Handbook of Hydrology, McGraw-Hill.

Nathan, R.J. and Bowles, D.S. (1997), A probability-neutral approach to the estimation of design snowmelt floods. Hydrology and Water Resources Symposium: Wai-Whenua, November 1997, Auckland, pp: 125-130.

Press, W.H., Teukolsky, S.A., Vetterling, W.T., Flannery, B.P. (1993), Numerical recipes in Fortran: the art of scientific computing. Cambridge University Press.

Robinson, K., Ling, F., Peterson, J., Nathan, R., Tye, I. (2012), Application of Monte Carlo Simulation to the Estimation of Large to Extreme Floods for Lake Rowallan Proceedings of the 34th Hydrology & Water Resources Symposium. ISBN 978-1-922107-62-6. Engineers Australia.

Saucier, R. (2000), Computer generation of statistical distributions. Army Research Laboratory ARL-TR-2168.

Scorah, M., Lang, S., and Nathan, R. (2015), Utilising the AWAP gridded rainfall dataset to enhance flood hydrology studies. Proc. 36th Hydrology and Water Resources Symposium, Hobart.

Vose, D. (2000), Risk analysis: a quantitative guide. John Wiley and Sons Ltd, Chichester.

Weinmann, P.E. and Nathan, R.J. (2004), The continuing challenge of estimating extreme floods from extreme design storms. Advances in Hydro-Science and Engineering, Volume VI, Proc. of the 6th Intern. Conf. on Hydro-Science and Engineering, Brisbane May 31 - June 3, 2004.

Weinmann, P.E., Rahman, A., Hoang, T.M.T., Laurenson, E.M. and Nathan, R.J. (2002), Monte Carlo Simulation of Flood Frequency Curves from Rainfall - The Way Ahead. Australian Journal of Water Resources, 6(1), 71-79.

Westra, S. (2012), Australian Rainfall and Runoff - Revision Project 18: Interaction of Coastal Processes and Severe Weather Events. ARR Report Number P18/S2/010, Engineers Australia.

Zheng, F., Westra, S., Sisson, S. and Leonard, M. (2014), Flood risk estimation in Australia's coastal zone: modelling the dependence between extreme rainfall and storm surge. Hydrology and Water Resources Symposium, Perth 2014.

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